Possibility of a continuous phase transition in random-anisotropy magnets with a generic random axis distribution

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Introduction

The structural disorder is inevitably present in many magnetic systems which undergo a phase transition. Of particular interest is its impact near the critical points, where even *weak disorder* can drastically modify the scaling behavior. For instance, in magnetic systems *weak quenched disorder* can change the characteristics of the second order phase transition, also it can modify the nature of this phase, producing spin-glass order.

Different types of structural randomness:

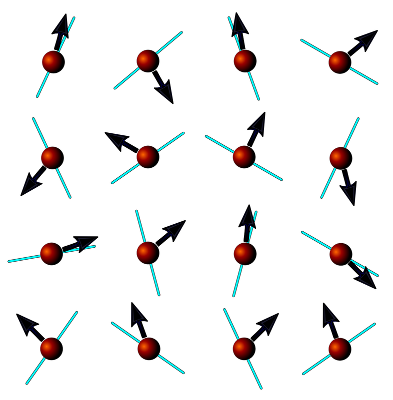
- random sites: magnetic phase transitions in crystalline alloys of uniaxial magnets and their non-magnetic isomorphs;
- random fields: magnetically dilute crystals with applied uniform fields;
- random anisotropy: amorphous alloys of rare-earth compounds with aspherical electron distributions and transition metals.

In this work we analyse *the critical properties* of magnetic systems described by **the random-anisotropy model** (RAM) [1].

Our goal here is *a verification* of *an absence/presence* of a long-range ordered phase considering **random anisotropy disorder** with *a generic trimodal random axis distribution* [2].

Random anisotropy

System of interacting m-component spins $\vec{S}_{\mathbf{R}}$, located on sites of the d-dimensional hypercubic lattice [1]:



$$\mathcal{H} = -\sum_{\mathbf{R},\mathbf{R}'} J_{\mathbf{R},\mathbf{R}'}\vec{S}_{\mathbf{R}}\vec{S}_{\mathbf{R}'} - D\sum_{\mathbf{R}}(\hat{x}_{\mathbf{R}}\vec{S}_{\mathbf{R}})^2 \quad (1)$$

where $J_{\mathbf{R},\mathbf{R}'} = J > 0$ is a short-range ferromagnetic interaction between m-component spins $\vec{S}_{\mathbf{R}}$ and $\vec{S}_{\mathbf{R}'}$; $\hat{x}$ is a random unit vector indicating the direction of the local anisotropy axis on each site; and $D > 0$ is anisotropy strength.

$D/J \to \infty$ – majority of studies predicts spin-glass;

$D/J \sim 1$ – the final answer is not received.

low temperature phase and transition to it?

- MF suggests a ferromagnetism, but studies involving fluctuations yield different results;
- MC suggests spin glass and quasi-long-range ordering, but recent research → a ferromagnetic order.

Previous results

Isotropic distribution – the random vector $\hat{x}_{\vec{R}}$ is directed in any direction in m-dimensional hyperspace with equal probability:

$$p_i(\hat{x}) \equiv \left(\int d^m\hat{x}\right)^{-1} = \frac{\Gamma(m/2)}{2\pi^{m/2}}$$

Results: the continuous phase transition is absent.

Cubic distribution – the random vector $\hat{x}_{\vec{R}}$ is allowed to be directed along one of the $2m$ axes of the hypercubic lattice:

$$p_c(\hat{x}) = \frac{1}{2m}\sum_{i=1}^{m}\left\{\delta^{(m)}(\hat{x}-\hat{k}_i) + \delta^{(m)}(\hat{x}+\hat{k}_i)\right\},$$

where $\hat{k}_i, \dots, \hat{k}_m$ are unit vectors along the axes.

Prediction: the continuous phase transition not found, but solutions are reminiscent of the disordered Ising model.

[3] – at the one-loop order of perturbation theory (**RG**);

[4] – at the two-loop order of perturbation theory (**RG**);

[5] – at the five-loop order of perturbation theory (**RG**).

Effective Hamiltonians

We map the spin lattice model (1) onto an effective φ^4 theory using *the Hubbard-Stratonovich transformation* and averaging over quenched disorder encoded by the local random vectors $\{\hat{x}_{\vec{R}}\}$ (all directions are fixed).

- **"Pure" system**

$$F = -\frac{1}{\beta}\ln\mathcal{Z}$$

$$\mathcal{Z} = \sum_{\{S\}} e^{-\beta\mathcal{H}} \quad \to \quad \mathcal{Z} = \int d\vec{\phi}\, e^{-\beta\mathcal{H}_{eff}}$$

$$\mathcal{H} = -\sum_{\mathbf{R},\mathbf{R}'} J_{\mathbf{R},\mathbf{R}'}\vec{S}_{\mathbf{R}}\vec{S}_{\mathbf{R}'} \quad \to$$

$$\mathcal{H}_{eff} = -\int d^dR\left\{\frac{1}{2}\left[\mu_0^2|\vec{\phi}|^2 + |\vec{\nabla}\vec{\phi}|^2\right] + \frac{u_0}{4!}|\vec{\phi}|^4\right\}$$

- **Disorder** (we use *the replica trick*)

$$F = -\frac{1}{\beta}\overline{\ln\mathcal{Z}} = -\frac{1}{\beta}\lim_{n\to\infty}\frac{\overline{\mathcal{Z}^n - 1}}{n}$$

(the properties of the original system at $n \to 0$)

- isotropic distribution: $p_i(\hat{x}) = \frac{\Gamma(m/2)}{2\pi^{m/2}}$

$$\mathcal{H}_{eff} = -\int d^dR\left\{\frac{1}{2}\left[\mu_0^2|\vec{\phi}|^2 + |\vec{\nabla}\vec{\phi}|^2\right] + \frac{u_0}{4!}|\vec{\phi}|^4 + \frac{v_0}{4!}\sum_{\alpha=1}^{n}|\vec{\phi^\alpha}|^4 + \frac{z_0}{4!}\sum_{\alpha,\beta=1}^{n}\sum_{i,j=1}^{m}\phi_i^\alpha\phi_j^\alpha\phi_i^\beta\phi_j^\beta\right\}, \quad (2)$$

where $u_0 > 0$, $v_0 > 0$, $z_0 < 0$, $z_0/u_0 = -m$;

- cubic distribution: $p_c(\hat{x}) = \frac{1}{2m}\sum_{i=1}^m\left\{\delta^{(m)}(\hat{x}-\hat{k}_i) + \delta^{(m)}(\hat{x}+\hat{k}_i)\right\}$

$$\mathcal{H}_{eff} = -\int d^dR\left\{\frac{1}{2}\left[\mu_0^2|\vec{\phi}|^2 + |\vec{\nabla}\vec{\phi}|^2\right] + \frac{u_0}{4!}|\vec{\phi}|^4 + \frac{v_0}{4!}\sum_{\alpha=1}^{n}|\vec{\phi^\alpha}|^4 + \frac{w_0}{4!}\sum_{\alpha,\beta=1}^{n}\sum_{i=1}^{m}(\phi_i^\alpha)^2(\phi_i^\beta)^2 + \frac{y_0}{4!}\sum_{i=1}^{m}\sum_{\alpha=1}^{n}(\phi_i^\alpha)^4\right\}, \quad (3)$$

where $u_0 > 0$, $v_0 > 0$, $w_0 < 0$, $w_0/u_0 = -m$, and $\forall\, y_0$.

- generic trimodal random axis distribution: $p(\hat{x}) = q\,p_i(\hat{x}) + (1-q)\,p_c(\hat{x})$

$$\mathcal{H}_{eff} = -\int d^dR\left\{\frac{1}{2}\left[\mu_0^2|\vec{\phi}|^2 + |\vec{\nabla}\vec{\phi}|^2\right] + \frac{u_0}{4!}|\vec{\phi}|^4 + \frac{v_0}{4!}\sum_{\alpha=1}^{n}|\vec{\phi^\alpha}|^4 + \frac{w_0}{4!}\sum_{\alpha,\beta=1}^{n}\sum_{i=1}^{m}(\phi_i^\alpha)^2(\phi_i^\beta)^2 + \frac{y_0}{4!}\sum_{i=1}^{m}\sum_{\alpha=1}^{n}(\phi_i^\alpha)^4 + \frac{z_0}{4!}\sum_{\alpha,\beta=1}^{n}\sum_{i,j=1}^{m}\phi_i^\alpha\phi_j^\alpha\phi_i^\beta\phi_j^\beta\right\}, \quad (4)$$

where $u_0 > 0$, $v_0 > 0$, $w_0 < 0$, $z_0 < 0$, and $\forall\, y_0$ [6].

The effective model (4) can be also derived by considering:

- A more general local anisotropy axis distribution.

Moments of the distribution $p(\hat{x})$: $M_{ijkl} = \int d^N\hat{x}\, p(\hat{x})\,\hat{x}^i\hat{x}^j\hat{x}^k\hat{x}^l$.

If $M_{ij} = \frac{\delta_{ij}}{m}$, then the fourth moment is $M_{ijkl} = A(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + B\delta_{ij}\delta_{ik}\delta_{il}$, where parameters A and B, depend on the distribution $p(\hat{x})$ and satisfy Cauchy inequalities $A(m+2) + B \geq 1/m$ and $3A + B \geq 1/m^2$ [5].

- Single-ion anisotropy with $p_i(\hat{x})$:

$$\mathcal{H} = -\sum_{\mathbf{R},\mathbf{R}'} J_{\mathbf{R},\mathbf{R}'}\vec{S}_{\mathbf{R}}\vec{S}_{\mathbf{R}'} - D\sum_{\mathbf{R}}(\hat{x}_{\mathbf{R}}\vec{S}_{\mathbf{R}})^2 + V\sum_{\mathbf{R}}\sum_{i=1}^{m}(S_{\mathbf{R}}^i)^4,$$

where V is the cubic anisotropy strength

(?) **Some research with Hamiltonian (4) suggests the continuous phase transition ⇒ existence ferromagnetic order [7].**

The field-theoretical RG aproach

$$\delta\left(\sum_i^L p_i + \sum_j^N k_j\right)\mathring{\Gamma}^{(L,N)}(\{p\};\{k\};\mu_0^2;\{\mathring{\lambda}\}) = \int^{\Lambda_0} d^dR_1\dots d^dR_L d^dr_1\dots d^dr_N$$
$$\times e^{i(\sum p_iR_i + \sum k_jr_j)}\left\langle\phi^2(R_1)\dots\phi^2(R_L)\phi(r_1)\cdots\phi(r_N)\right\rangle_{1PI}^{\mathcal{H}_{eff}}, \quad (5)$$

where $\{\mathring{\lambda}\} = \{u_0, v_0, w_0, y_0, z_0\}$ are bare coupling constants, $\{p\}$, $\{k\}$ are external momenta, Λ_0 is a cut-off parameter, and μ_0 is a bare mass. Divergence in the limit $\Lambda_0 \to \infty$

Renormalization: $\mathring{\lambda}_i = \mu^\varepsilon\frac{Z_{\lambda_i}}{Z_\phi^2}\lambda_i,$

where μ is the renormalized mass in **the massive scheme** and the scale parameter in **the minimal subtraction scheme**; Z_{λ_i}, Z_ϕ and Z_{ϕ^2} are the renormalization factors.

Then the renormalized vertex functions do not contain divergence: $\Gamma_R^{(L,N)} = Z_{\phi^2}^L Z_\phi^{N/2}\mathring{\Gamma}^{(L,N)}$

- The RG functions: $\beta_{\lambda_i} = -\lambda_i(\varepsilon + \gamma_{\lambda_i} - 2\gamma_\phi)$

$$\gamma_{\lambda_i} = \frac{\partial\ln Z_{\lambda_i}}{\partial\ln\mu}\Big|_{\{\mathring{\lambda}\},\mu_0}, \quad \gamma_\phi = \frac{\partial\ln Z_\phi}{\partial\ln\mu}\Big|_{\{\mathring{\lambda}\},\mu_0}, \quad \overline{\gamma}_{\phi^2} = -\frac{\partial\ln\overline{Z}_{\phi^2}}{\partial\ln\mu}\Big|_{\{\mathring{\lambda}\},\mu_0}, \quad (6)$$

where $\varepsilon = 4 - d$ and $\overline{Z}_{\phi^2} = Z_{\phi^2}Z_\phi$. The β and γ functions characterize the change of the vertex functions under the RG transformation, and thus allow one to calculate the scaling behavior in the critical region controlled by a fixed point (FP)

$$\beta_{\lambda_i}(\{\lambda^*\}) = 0, \quad i = 1, 2, \dots. \quad (7)$$

The FP solution $\{\lambda^*\}$ of (7) describes the critical point of the system if it is **stable** and **accessible** from the initial conditions. The FP is stable if all the eigenvalues $\{\omega_i\}$ of the stability matrix at this point have the positive real parts: $B_{ij} = \partial\beta_{\lambda_i}/\partial\lambda_i|_{\lambda_i=\lambda_i^*}$

References

[1] R. Harris, M. Plishke, M. J. Zuckermann, Phys. Rev. Lett. **31**, 160 (1973)

[2] D. Shapoval, M. Dudka, A. A. Fedorenko, and Yu. Holovatch, Phys. Rev. B **101**, 064402 (2020)

[3] A. Aharony, Phys. Rev. B **12** 1038 (1975)

[4] M. Dudka, R. Folk, Yu. Holovatch, Condens. Matter Phys. **4**, 77 (2001); ibid. **4** 459 (2001)

[5] P. Calabrese, A. Pelissetto, E. Vicari, Phys. Rev. E **70**, 036104 (2004)

[6] D. Mukamel and G. Grinstein, Phys. Rev. B **25**, 381 (1982)

[7] V. Dubs, V. Prudnikov, and P. Prudnikov, Theor. Math. Phys. **190**, 359 (2017)

Results

Applying renormalization schemes we obtain in the two-loop approximation the RG-functions

$$\beta_{\lambda_i}, \gamma_{\lambda_i}, \gamma_\phi, \overline{\gamma}_{\phi^2}$$

- where we used the massive scheme;
- additionally we verified results in the minimal subtraction scheme;
- and also check that the RG functions satisfy the properties that follow from the original model (4) and reproduce properly the results known for reduced models [5].

Next applying resummation procedure (Padé-Borel technique) for the analysis of the RG functions (at the fixed $d = 3$) we obtain the sets of the FPs for $m = 2$ and $m = 3$ in the massive and the minimal subtraction schemes, respectively.

- **We have found that in the most general case of random anisotropy distribution there is no stable FP accessible from physical initial conditions, and thus, there is no continuous phase transition into a ferromagnetic state.**
- **However, the continuous phase transition can be observed for some particular distributions[4].**

Conclusions

- We have considered the influence of complex distributions, which leads to an effective functional of ϕ^4-theory with five terms of different symmetry.
- Working within field-theoretical renormalization group (RG) theory we have calculated corresponding two-loop RG functions in **the massive scheme** and **the minimal subtraction scheme**.
- We have verified that the RG functions reproduce the results known for the limiting cases of the *isotropic* and *cubic distributions*.
- Resumming them with help of Padé-Borel technique we have solved FP equations and analyzed stability of obtained solutions.
- Our results give no evidence of a FP which is simultaneously stable and accessible from the initial conditions, therefore indicating absence of continuous phase transition into ferromagnetic state.