Phase-ordering kinetics and persistence of the two-dimensional long-range Ising model at zero temperature

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Abstract

We investigate the phase-ordering kinetics of the $d = 2$ dimensional long-range Ising model with power-law decaying interactions $\propto 1/r^{d+\sigma}$. Recently, we have numerically confirmed that the characteristic length $\ell(t)$ after a quench to $0 < T < T_c$ grows as predicted by Bray and Rutenberg [1], i.e., for finite T the growth is σ *dependent*. We now perform a quench to $T = 0$, for which we observe that the growth exponent $\alpha \approx 3/4$ is *independent* of σ and different from $\alpha = 1/2$ as is known for the nearest-neighbor model. Additionally, we investigate the persistence of the local order parameter and provide estimates for the persistence exponent θ and the fractal dimension d_f of the persistent lattice. In the limit of large σ only the fractal dimension d_f of the nearest-neighbor Ising model is recovered, while θ differs significantly. This we understand from the unexpected value for α and a conjectured relation between these exponents, which we confirm numerically for the long-range model.

Model and Phase Ordering Kinetics

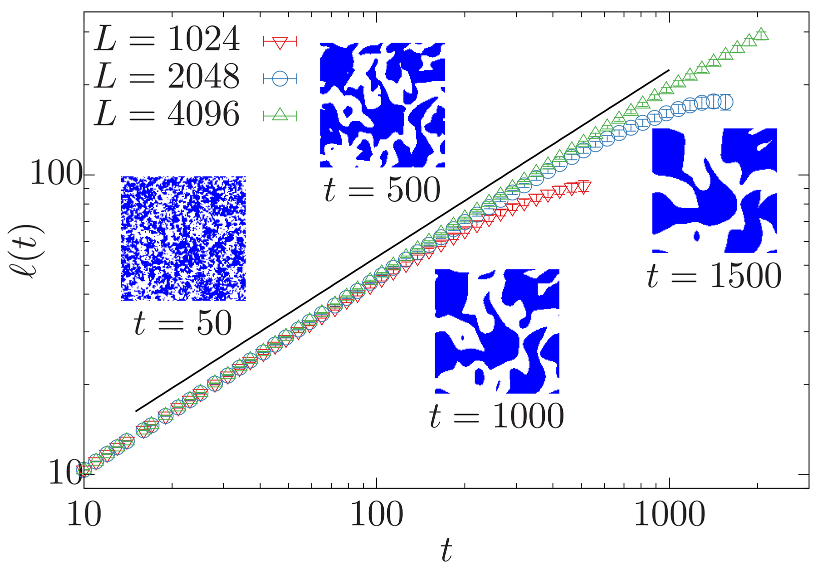
The long-range Ising model with power-law decaying potential can be described by the Hamiltonian

$$\mathcal{H} = -\frac{1}{2}\sum_i \sum_{j\neq i} J(r_{ij}) s_i s_j \text{ and } J(r_{ij}) = \frac{1}{r_{ij}^{d+\sigma}}$$

where the spins $s_i = \pm 1$ are placed on a square lattice.
In phase ordering kinetics, starting from a disordered configuration, this system is then quenched to $0 \neq T < T_c$ and the ordering of the system is investigated. For this model, there exists a prediction for the characteristic length during this process [1]:

$$\ell(t) \propto t^\alpha = \begin{cases} t^{\frac{1}{1+\sigma}} & \sigma < 1 \\ (t \ln t)^{1/2} & \sigma = 1 \\ t^{\frac{1}{2}} & \sigma > 1 \end{cases}$$

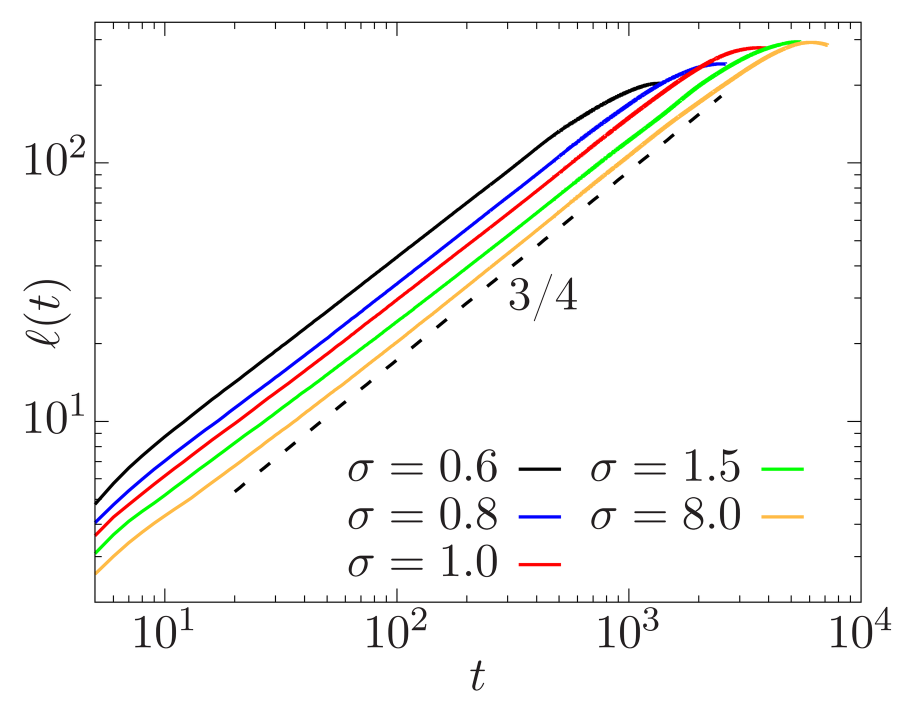
For $\sigma > 1$ one thus sees short-range like behavior, for $\sigma < 1$ the growth becomes σ dependent. We have shown this for the first time numerically in Ref. [2]; see also Ref. [3].



$\ell(t)$ for $\sigma = 0.6$
solid black line is prediction

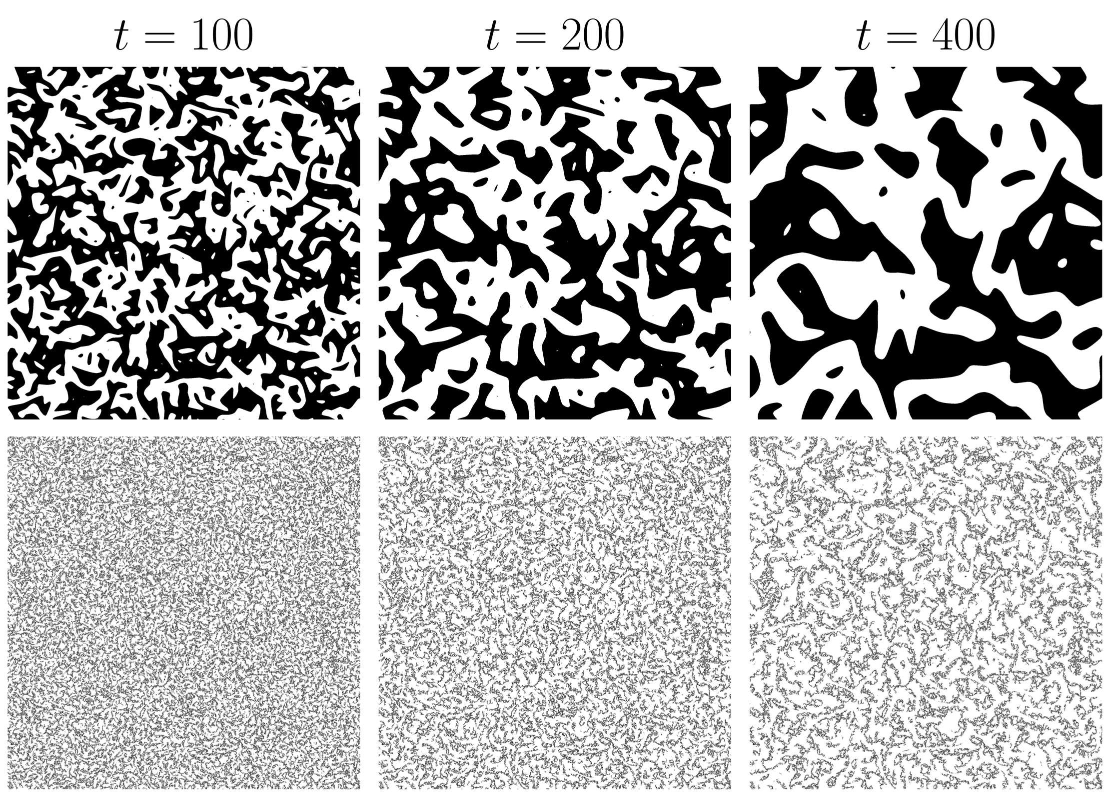
Zero Temperature

We now quench to $T = 0$ and do not observe the same growth law anymore [4, 5]:



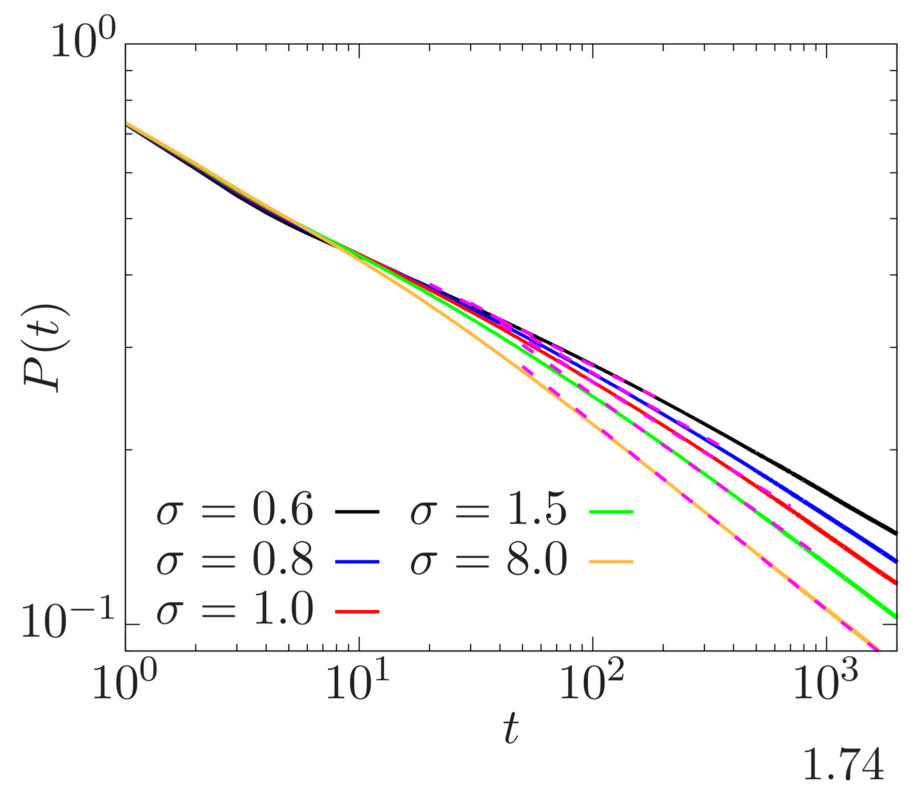
$\ell(t)$ for different σ when quenched to $T = 0$.

This appears puzzling on first sight, but can qualitatively be understood from $d = 1$, where for $T = 0$ ballistic growth independent of σ is observed [6].



Persistence: Interested in the spins which have never flipped until time t during the phase ordering protocol. Upper row: Direct lattice snapshots. Lower row: Persistent lattice snapshots.

Persistence Probability and Fractal Dimension



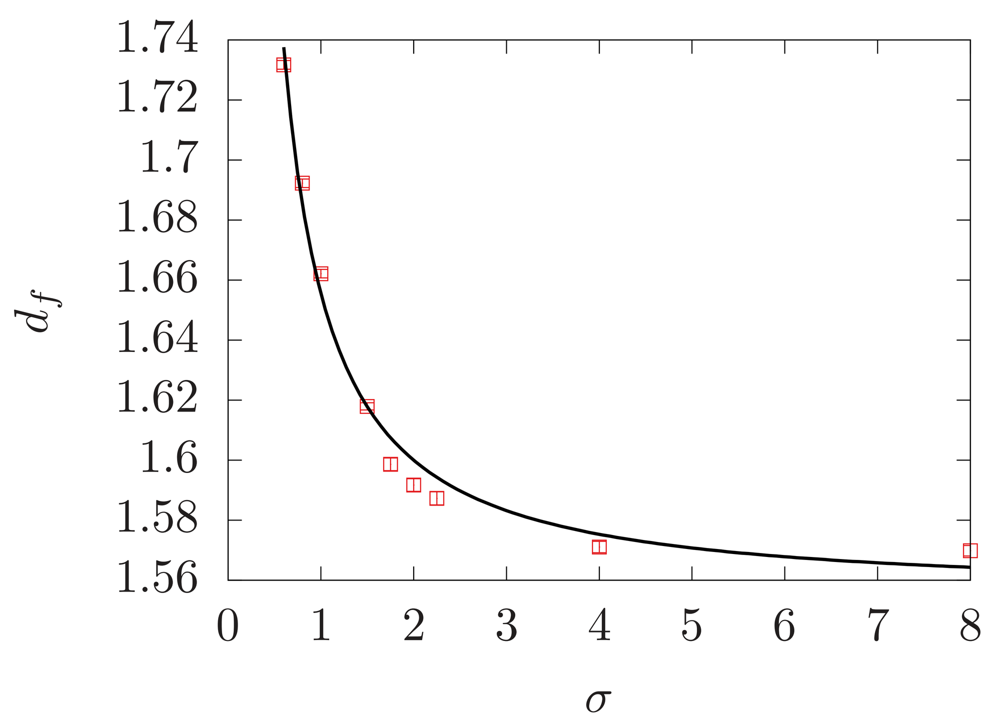
The persistence probability $P(t)$ of spins which have never flipped often decays as a power law $P(t) \sim t^{-\theta}$ with the nontrivial exponent θ.

Here, this is also the case with a clearly σ dependent exponent θ.

Fractal dimension measured from the correlation function of the persistent lattice $D(r, t)$ as

$$D(r,t)/P(t) \sim \begin{cases} x^{-\kappa} & x \ll 1 \\ 1 & x \gg 1 \end{cases}$$

where $\kappa = d - d_f$ with the fractal dimension d_f.



Approaches the value of the nearest-neighbor model value of $d_f \approx 1.57$ for $\sigma \to \infty$.

To get an idea of the functional dependency of $d_f(\sigma)$, we fitted a power law of the form $d_f(\sigma) = d_{f,\infty} + A\sigma^{-B}$ to the data of d_f, where $d_{f,\infty} = 1.55\bar{5}$ is the estimate of d_f obtained by assuming $\alpha = 3/4$ and $\theta = 1/3$, which is very close to the fitted value for $\sigma = 8$. The corresponding fit is plotted above and has $A = 0.1001(7)$ and $B = 1.17(2)$.

There exists a proposed relationship reading $d - d_f = \theta/\alpha$ [7], which is checked in the plot of $P(t)$ by taking the measured d_f and $\alpha = 3/4$ and plugging them into this equation to obtain θ. This is plotted as dashed magenta lines which fit exceptionally well.

References

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Conclusion

We have studied the zero-temperature coarsening of the two-dimensional long-range Ising model with non-conserved order parameter by tuning the degree of the long-range interactions via the power-law exponent σ. It is found that the growth exponent $\alpha \approx 3/4$ is independent of σ. For our most short-range-like case of $\sigma = 8$, we find that the fractal dimension is compatible with the value found for the nearest-neighbor Ising model and reads $d_f \approx 1.57$. Evidence was provided in favor of the relation $d - d_f = \theta/\alpha$, which relates the nonequilibrium exponents. We investigate a range of different σ and find that d_f (and thereby θ) varies continuously with σ. After completion of this work, we became aware of the very recent reference [5] where the authors focus on the growth exponent α and also find $\ell(t) \sim t^{3/4}$ independent of σ.