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Loss of synchronization in locally coupled one dimensional oscillator systems

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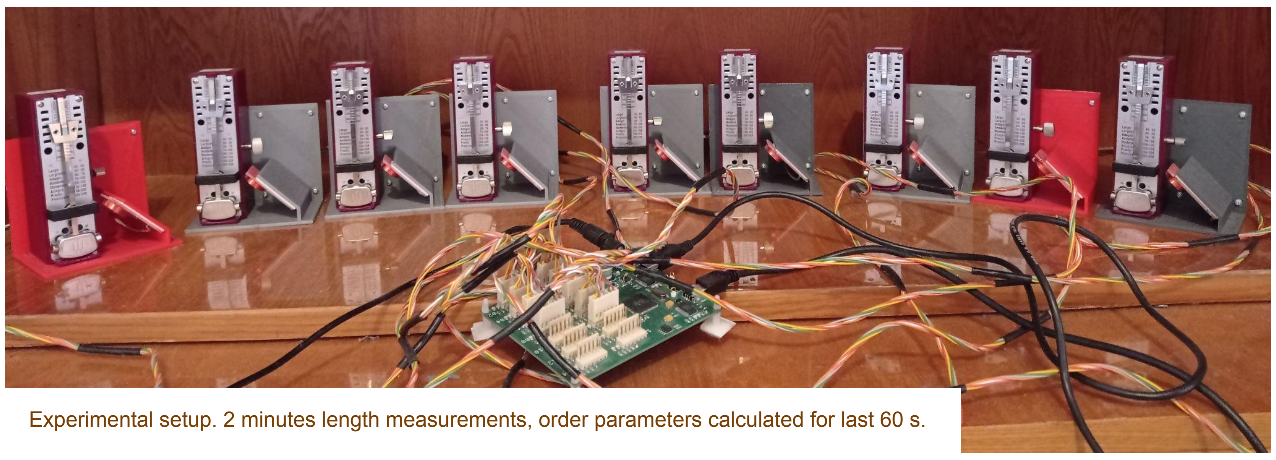
Synchronization in one-dimensional oscillator systems are studied both experimentally and by computer simulations. The oscillators have a spread in their natural frequencies and are arranged in a ring-like topology, interacting only with their nearest neighbors. According to the well-known Mermin-Wagner theorem in the thermodynamic limit the system should not be able to synchronize. Our aim here is to investigate how the synchronization order parameter vanishes as the system size is increased.

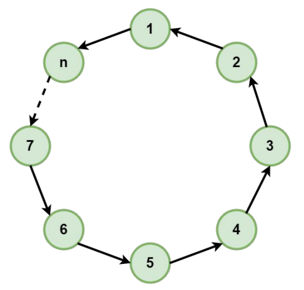
Experimental setup

- Electronically coupled mechanical metronomes are considered.
- Magnetic braking force acts when the limb of the metronomes reached the maximum amplitude.
- Braking of each metronome is governed by the phase of it's neighbor.
- If the metronome's period is shorter than the one of it's reference, the limb's motion is delayed by magnetic braking.
- Multiple measurements were done with closed 1D loops containing from 2 up to 9 metronomes.

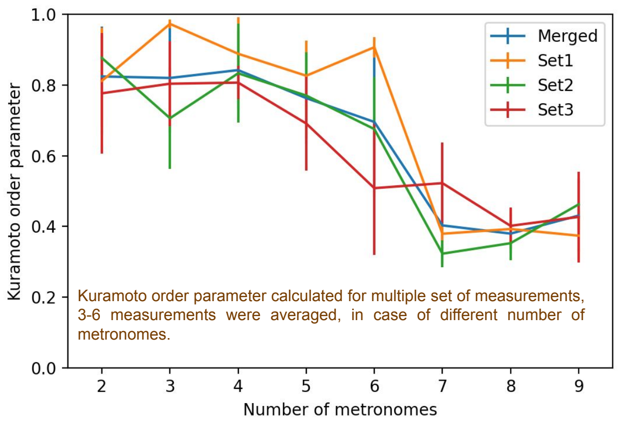


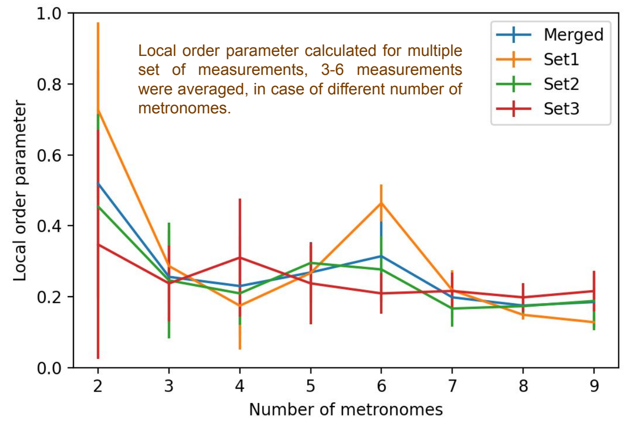
Experimental setup. 2 minutes length measurements, order parameters calculated for last 60 s.

System topology

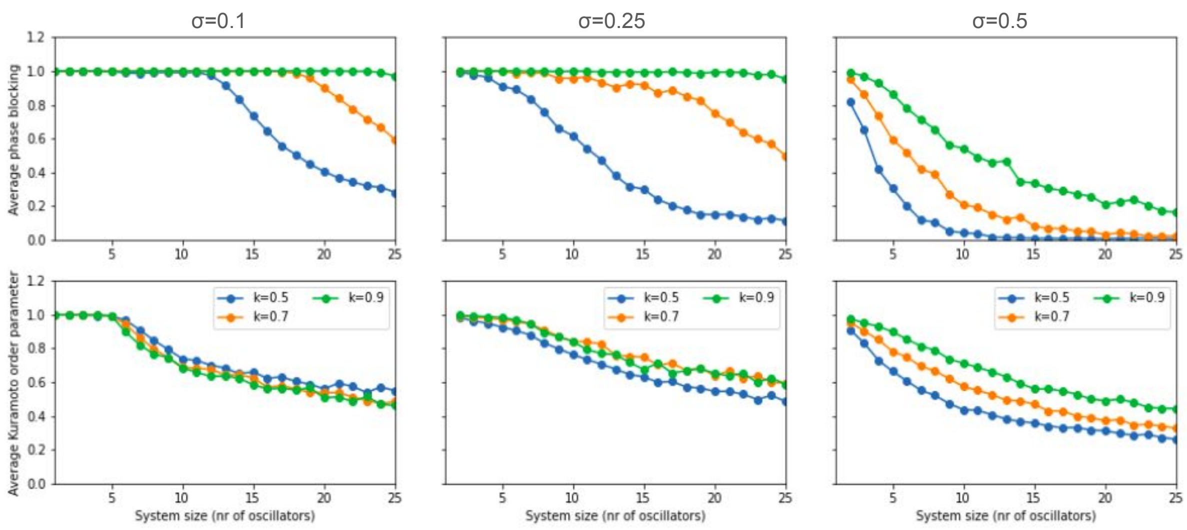


As a function of the number of metronomes in the loop synchronization of the system is studied.
Two type of sync order parameters were determined:
- **global** Kuramoto order parameter (based on the estimated phases of the metronomes)
- **local** order parameter (based on the phase-locking of the neighbouring oscillators)





Kuramoto model approach [1]



Numerical simulation results for N ∈ [2,25] number of oscillators; coupling constant K ∈ {0.5, 0.7, 0.9}; angular velocity centered to ω_0= 6.28 rad/s, with a Gaussian distribution for σ∈ {0.1, 0.25, 0.5}. Time step set to dt = 0.01 s, statistics made for st=200 independent simulations, in which initial ω were randomly chosen in the given interval for the N oscillators. The order parameters were calculated afters teps = Nx1000 thermalization steps.

Locally coupled Kuramoto chain

$$\dot{\theta}_i = \omega_i + K\sum sin(\theta_i - \theta_{i-1})$$

Kuramoto order parameter (global)

$$re^{i\psi} = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$$

Local order parameter

- determine the phase differences between neighboring oscillators at a given time T_0
- run the simulation for definite number of steps **n** and determine the phase differences at T_1= T_0+ **n**·dt
- compare the phase differences at T_1 with those measured at T_0
  - if the differences are below a given threshold, then assign 1
  - otherwise assign 0
- sum up the assigned values and divide by the number of oscillators
- averaging over time

Realistic model of metronomes [2]

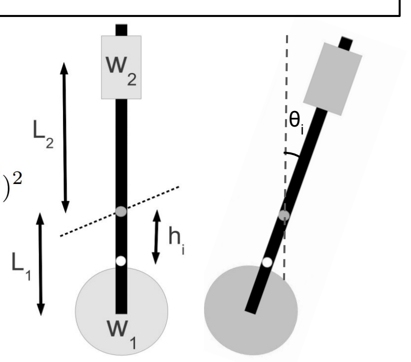
Computational model is based on a realistic mechanical model of the metronomes with an interaction mechanism closely resembling the experimentally implemented one. We solved numerically the involved evolution equations and determined the same synchronization order parameters as the ones in the experiments.
Coupling realized in similar manner with the braking mechanism in the experiments.

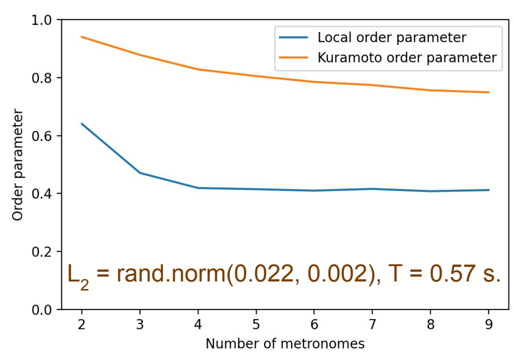
$$\ddot{\theta}_i = \frac{M_i - mgh_i sin\theta_i - c_\theta \dot{\theta}_i}{mh_i^2 + J_i}$$

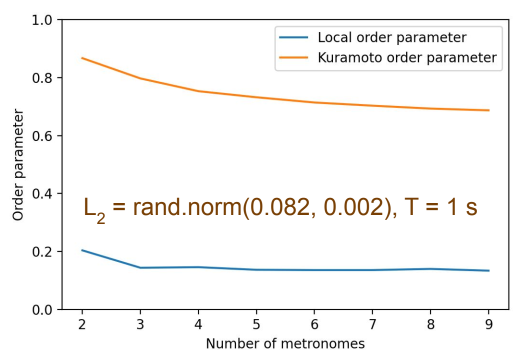
, where

$$h_i = \frac{w_1 L_1 - w_2 L_2^{(i)}}{w_1 + w_2} \quad \text{and} \quad J_i = w_1 L_1^2 + w_2 (L_2^{(i)})^2$$

$$\mathbb{M}_i = \begin{cases} M/dt & \text{if } \theta_i(t-dt) < 0 \text{ and } \theta_i(t) > 0 \\ -M/dt & \text{if } \theta_i(t-dt) > 0 \text{ and } \theta_i(t) < 0 \\ 0 & \text{in any other case} \end{cases}$$







Simulation results using the equation of motion of metronomes:
Parameters:
w_1= 0.025 kg, w_2= 0.0069 kg,
L_1 = 0.0358 m,
M = $9\text{x}10^{-5}$, c_θ = 10^{-4} braking coeff.
Averaging over 500 simulations, timestep dt = 0.001s, θ_i were randomly initialized.

Conclusions

Experiments and computational models yield similar results for the monotonic decaying trend of the synchronization order parameters as a function of the system size. This is in agreement with one would expect from the Mermin-Wagner theorem [3]. The simple locally coupled Kuramoto model is already able to describe qualitatively the behavior of the experimental interacting metronomes.

References
[1] Y. Kuramoto, "Self-entrainment of a population of coupled non-linear oscillators," International Symposium on Mathematical Problems in Theoretical Physics, vol. 39, p. 420–422, 1975.
[2] Sz. Boda et. al., "The rhythm of coupled metronomes", The European Physical Journal B, vol. 86, 2013.
[3] N. D. Mermin and H. Wagner, "Absence of Ferromagnetism or Antiferromagnetism in One- or Two-Dimensional Isotropic Heisenberg Models", Physical Review Letters, vol. 17, Issue 22, pp. 1133-1136, 1966.