Loss of synchronization in locally coupled one dimensional oscillator systems

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Synchronization in one-dimensional oscillator systems are studied both experimentally and by computer simulations. The oscillators have a spread in their natural frequencies and are arranged in a ring-like topology, interacting only with their nearest neighbors. According to the well-known Mermin-Wagner theorem in the thermodynamic limit the system should not be able to synchronize. Our aim here is to investigate how the synchronization order parameter vanishes as the system size is increased.

**Experimental setup**

- Electronically coupled mechanical metronomes are considered.
- Magnetic braking force acts when the limb of the metronomes reached the maximum amplitude.
- Braking of each metronome is governed by the phase of its neighbor.
- If the metronome’s period is shorter than the one of its reference, the limb’s motion is delayed by magnetic braking.
- Multiple measurements were done with closed 1D loops containing from 2 up to 9 metronomes.

![System topology](image)

As a function of the number of metronomes in the loop, synchronization of the system is studied. Two type of sync order parameters were determined:
- Global Kuramoto order parameter (based on the estimated phases of the metronomes)
- Local order parameter (based on the phase-locking of the neighbouring oscillators)

**Kuramoto model approach**

\[
\dot{\theta}_i = \omega_i + K \sum_{j \neq i} \sin(\theta_i - \theta_j)
\]

**Realistic model of metronomes**

Computational model is based on a realistic mechanical model of the metronomes with an interaction mechanism closely resembling the experimentally implemented one. We solved numerically the involved evolution equations and determined the same synchronization order parameters as the ones in the experiments. Coupling realized in similar manner with the braking mechanism in the experiments.

\[
\begin{align*}
\dot{\theta}_i &= M_i - \omega_i^2 \sin \theta_i - c \dot{\theta}_i \\
M_i &= M_0 + J_i
\end{align*}
\]

, where

\[
\begin{align*}
h_i &= w_i L_i - w_i \frac{L_i^2}{L_i^2 + L_{i+1}^2} \\
J_i &= w_i^2 L_i^2 + w_i L_i L_{i+1} \frac{L_i^2 + L_{i+1}^2}{L_i^2 + L_{i+1}^2}
\end{align*}
\]

**Conclusions**

Experiments and computational models yield similar results for the monotonic decaying trend of the synchronization order parameters as a function of the system size. This is in agreement with one would expect from the Mermin-Wagner theorem [3]. The simple locally coupled Kuramoto model is already able to qualitatively behave the experimental interacting metronomes.

**References**