Failure avalanches on complex networks

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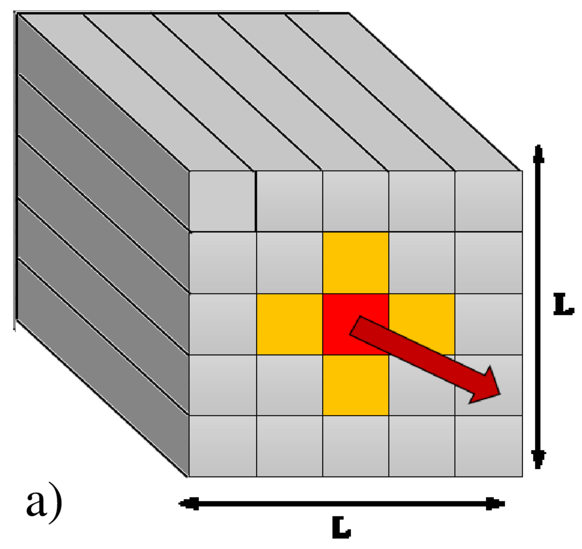
Introduction

The fracture of heterogeneous materials under a slowly increasing external load proceeds in avalanches of local failure events: the cracking of a material region can trigger additional failure events due to the subsequent redistribution of mechanical load over the intact elements [1]. The range of load sharing following local breaking and the degree of disorder of the strength of individual elements of the system play a crucial role in the evolution of failure avalanches. Here we investigate how the network structure of load sharing connections affects the dynamics of avalanche propagation [2,3].

Model construction

We use the fiber bundle model (FBM) of heterogeneous materials [2] to generate quasi-static failure processes.

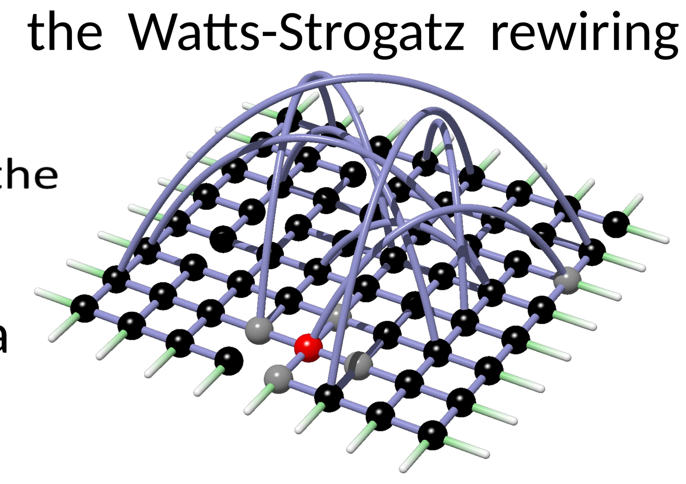
- In FBMs the sample is discretized in terms of parallel fibers
- Fibers:
  - loaded parallel to fibers' direction
  - show perfectly brittle behavior
  - have the same elastic modulus E
  - have random breaking threshold σ_{th}



- Equal load sharing (ELS): The excess load after failure is equally shared by all the intact fibers.
- Local load sharing (LLS): The excess load is shared by the nearest neighbors → complex stress field

Network of load sharing connections

- Starting from a regular square lattice of fibers, networks of load sharing connections are generated with the Watts-Strogatz rewiring method [5].
  - The rewiring probability takes values in the range $0 \leq P \leq 1$.
- An avalanche, triggered by the failure of a single fiber, has a size , and a duration .

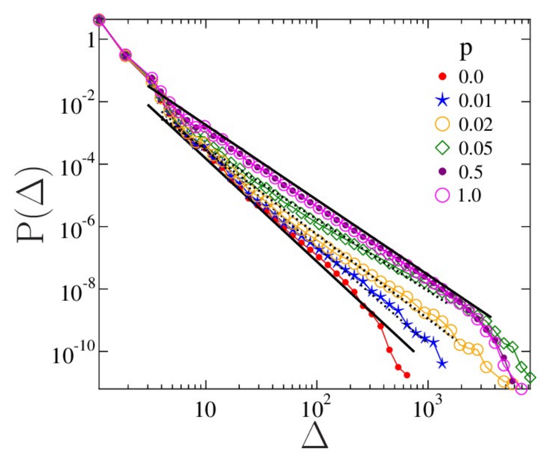


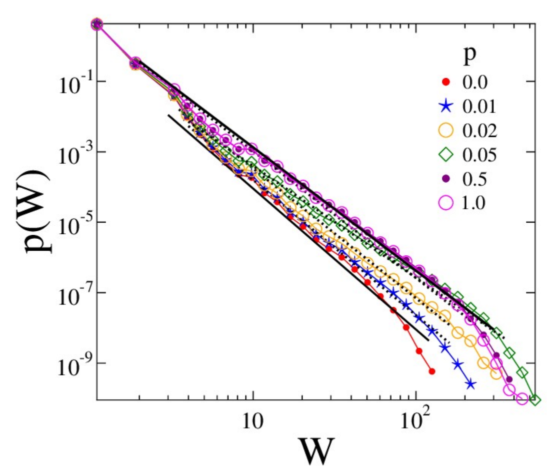
AVALANCHE STATISTICS

- The propablity distribution of size ρ(Δ) and duration ρ(W) of bursts have power law behaviour

$$\rho(\Delta) = \Delta^{-\tau} \text{ and } \rho(W) = W^{-\tau_w}$$

However tha exponent τ_w and τ both depend on the rewiring probblity P i.e, the exponent decreases with increasing P.



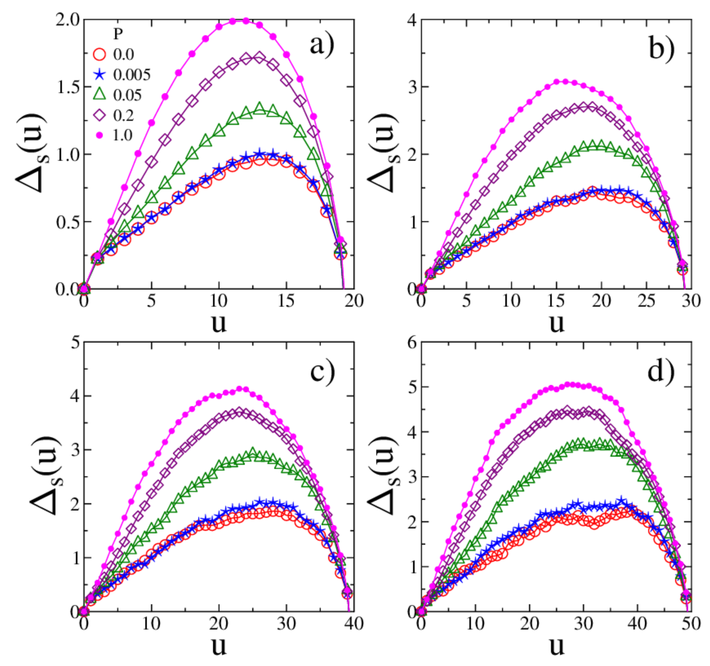


- In the limt of p =0, the results agree with LLS avalanche statstics on a square lattice ($\tau \approx 3.5$ and $\tau_W = 4.0$) [4].

However, for p=1 where the connections from a random graph, the exponents are smaller than their mean field counter parts, ($\tau \approx 5/2$ and $\tau_W = 7/2$) [2, 3, 4].

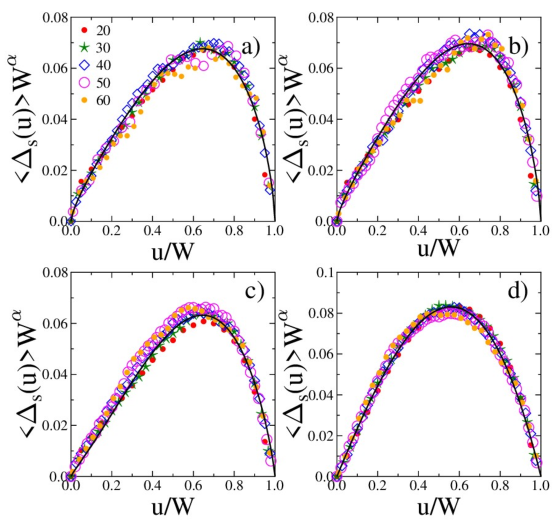
AVERAGE TEMPORAL PROFILE OF AVALANCHES

- For low P the profiles have a strong right handed asymmetry at all durations W. It shows that avalanches start slowly, gradually accelerate, and suddenly stop.
- As P increases the degree of asymmetry decreases but even at $P = 1$ some asymmetry remains.



- Bursts of a longer duration W have a larger average height and average size .

Scaling behaviour of pulse profile

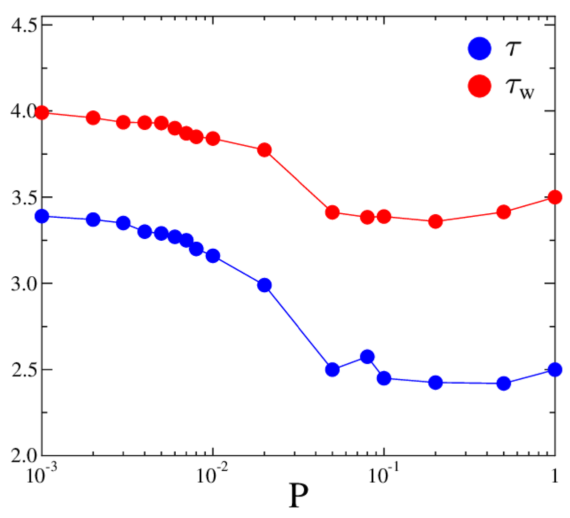


- Profiles for a fixed P with different durations collapsed on each other by rescaling with an apropriate power of W.
- The continuous lines represent fits of the scaling function with the equation

$$f(x) = [x(1-x)]^{\alpha}\left[1 - a\left(x - \frac{1}{2}\right)\right]$$

The value of exponent α starts from 0.68 for p=0 and approaches 1 at p=1. the parameter a captures the asymmetry of the scaling curves. Due to the right handed asymmetry, it has a negetive value, starting from a = - 0.9 (P = 0) to a =- 0.5 (P = 1).

Characteristic exponents



- We determined the value of the characteristic exponents of the system as a function of the rewiring probability .
  - The analysis showed that the system has two phases: at sufficiently low values of P the statistics and dynamics of avalanches coincides with LLS FBMs.
- At high Ps a distinct behaviour emerges where the exponents are close to the ELS (mean field) exponents of FBMs, however, they are still smaller. The two phases are separated by a broad transition regime.

Discussion

Computer simulations revealed that already a very small fraction of long range connections is sufficient to change the dynamics of avalanche propagation and the statistics of global avalanche characteristics. Deviations from the LLS behavior starts already at $P \approx 0.01$.

On random graphs $P = 1$ the dynamics of avalanche propagation is very similar to that of ELS FBMs (mean field limit), where all fibers share the same load and no stress fluctuaations can arise.

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References

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