Gravity driven fragmentation of a cohesive granular column

Csanád Szuszik and Ferenc Kun

Department of Theoretical Physics, University of Debrecen
H-4002 Debrecen, P. O. Box 400, Hungary

Motivations

Natural catastrophes like landslides are often caused by the nucleation and propagation of fractures in heterogeneous materials. Landslides are typically initiated by heavy raining events when water penetrates the pores and reduces the cohesion of soils leading to instability and cracking. When it happens on a steep slope, the moving mass could break up into pieces and the landslide gives rise to a debris flow composed of rapidly traveling fragments of soil and rocks [1]. Such devastating catastrophes endanger the infrastructure and take thousands of lives every year. In order to understand the emergence of landslides and debris flows we investigated the collapse of a granular column under the action of gravity by means of discrete element simulations [2].

Discrete element model of moistened soil piece

- In the model, a cylindrical sample of soil is represented as a random packing of spherical particles (Fig. 1).
- The random particle packing is generated through sedimentation of spherical particles in a container of cylindrical shape, using the Hertz contact law (Fig. 2). The radius of particles is sampled from a log-normal distribution. Simulations were performed with specimens of about 50.000 particles.
- Cohesion is introduced by connecting the particles with non-linear spring elements. The constitutive law of springs captures the linearly elastic behaviour of particle contacts at small deformations, the plasticity beyond a yield threshold, and the gradual softening and final breaking at large separation distances (Fig. 4).

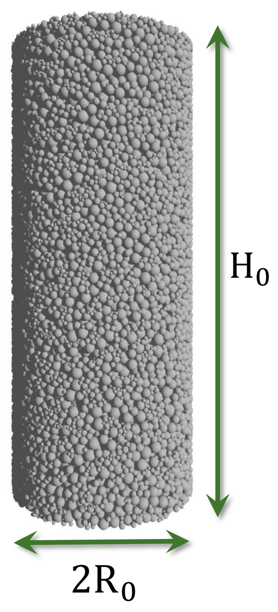
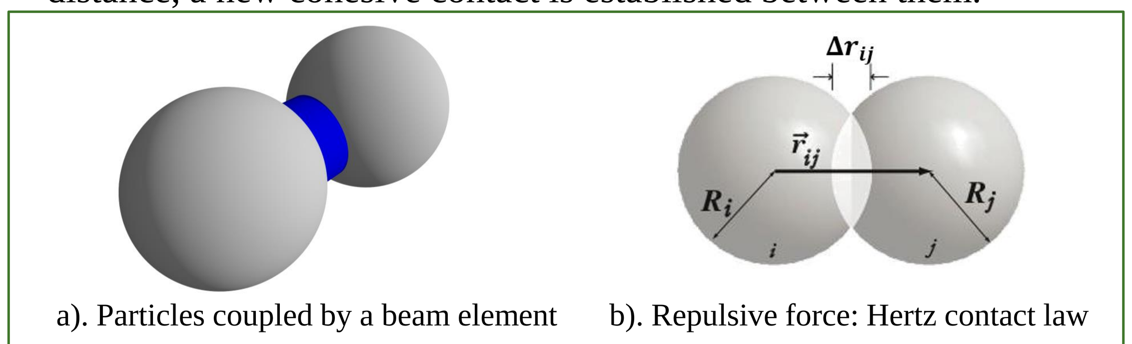


Fig. 1. Cylindrical soil sample generated by a sedimentation algorithm.

- The most important feature of the interaction is that particle contacts can be healed, i.e. if two particles approach each other within a capture distance, a new cohesive contact is established between them.



a). Particles coupled by a beam element b). Repulsive force: Hertz contact law

Fig. 2. Interactions between particles.

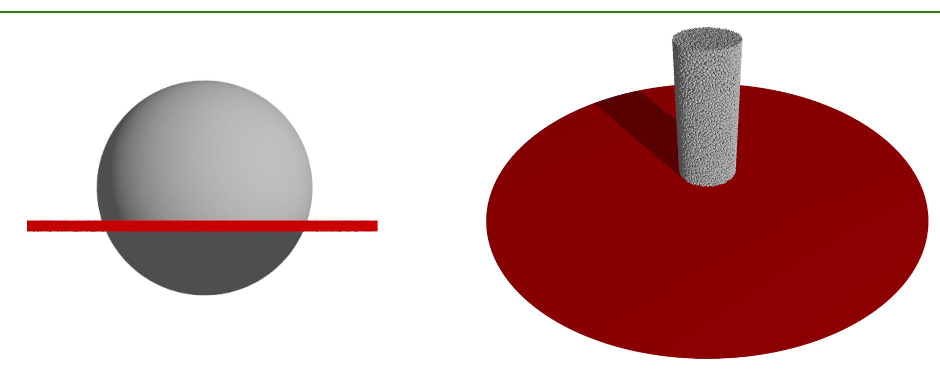


Fig. 3. The ground under the sample is represented by a hard plan.

- The interaction's non-linear shape and the braking-healing mechanism allows the model to create a simple qualitative picture of the moistened soil's behaviour.
- The cohesive interaction that we used is the approximation of the real one in a moistened soil, that has been determined by field and laboratory measurements. The approximation was used to reduce the number of parameters of the model, saving us a great amount of computing time during the simulations (Fig. 5.).
- The hard ground was represented by an underlying plane (Fig. 3.). The interaction with this wall is a simple linear repulsive force which depends on the degree of overlap, where friction was neglected.
- Computer simulations were performed varying the strength of cohesion (Y_k) in a broad range, while all other parameters of the model had been fixed.

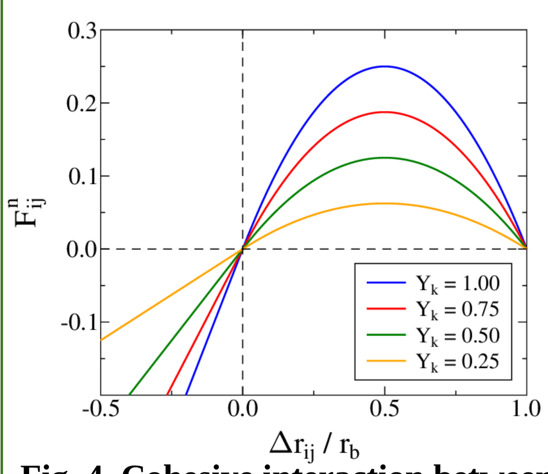


Fig. 4. Cohesive interaction between two particles.

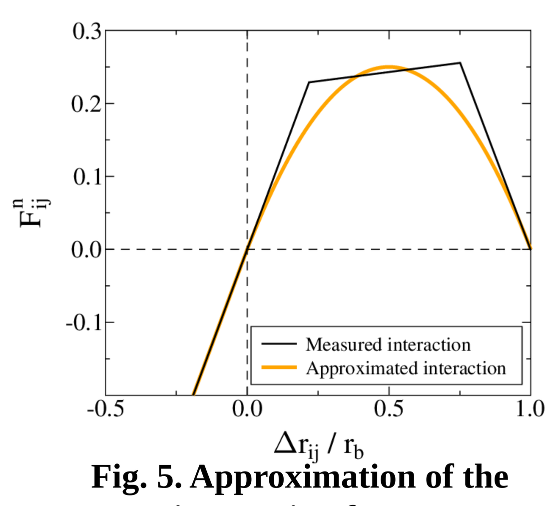


Fig. 5. Approximation of the interaction force.

The evolution of collapse

Our calculations revealed that at high cohesion Y_k the granular column sinks in but keeps its integrity (Fig. 6.). When the cohesion is week, the process of collapse cannot stop: the system breaks up into a large number of fragments (Fig. 7.). The two phases of high and low cohesion represent the landslide and the debris flow states, respectively.

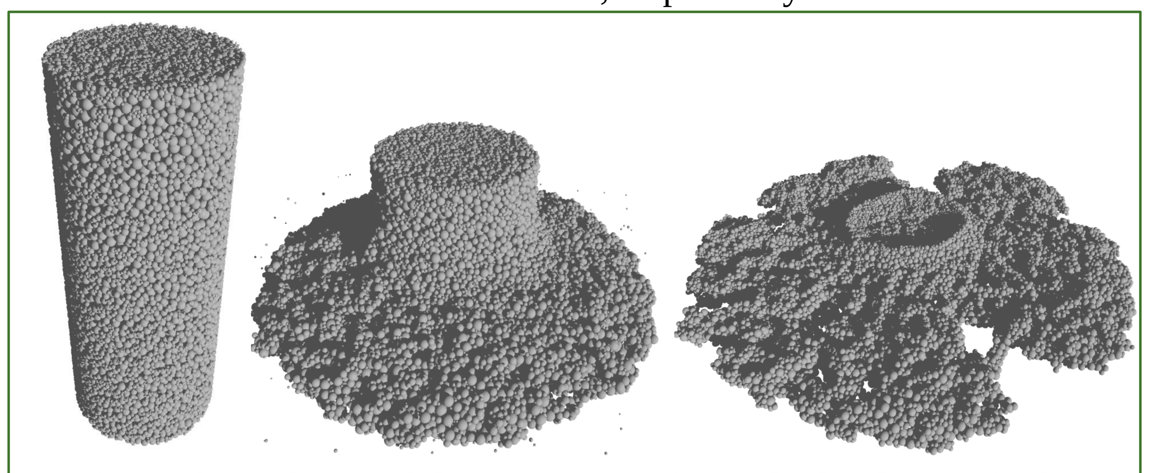


Fig. 6. The evolution of the system over time at high cohesion, in the landslide phase. The granular column on the left gradually sinks in (middle), keeping its integrity in the final state (right).

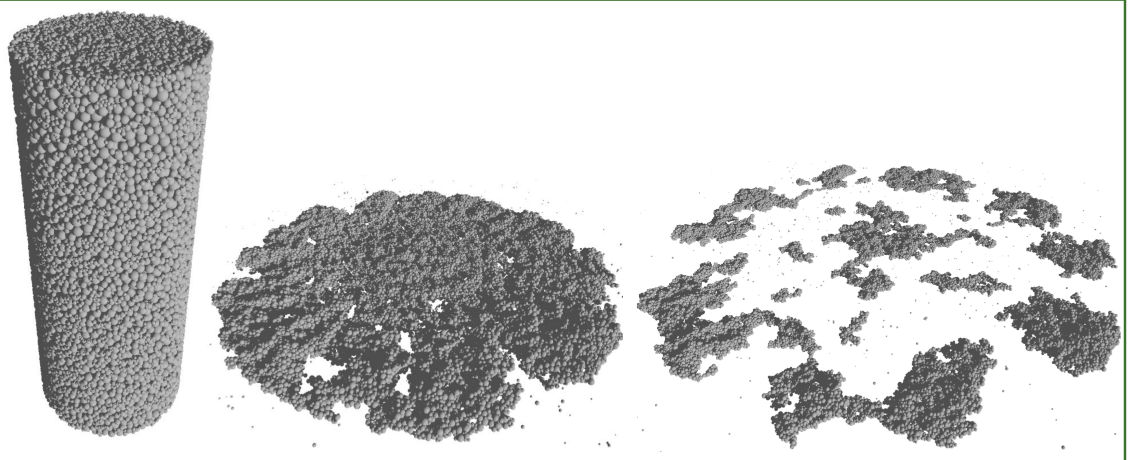


Fig. 7. The evolution of the system over time at lower cohesion, in the debris flow phase. Fragments run out at a high speed.

The two phases are separated by a Y_k^c critical value of the cohesion strength: for high cohesions $Y_k > Y_k^c$ the specimen sinks but eventually it comes to rest, where all kinetic energy is dissipated by the breaking-healing mechanism of particle contacts. In the final state the height h of the specimen is reduced from its initial value H_0, while on the underlying plane it spreads horizontally over a circular area of radius r_{max}. Consequently, some part of the initial potential energy is stored in deformation of the system. In the low cohesion phase $Y_k < Y_k^c$ a finite fraction of the energy remains stored in the motion of the fragments, because friction was neglected.

The transition between phases

Approaching the critical cohesion from the debris flow phase, the final state kinetic energy E_k^a converges to zero, because in the landslide phase all of the kinetic energy is dissipated by the breaking-healing mechanism. We found that this convergence has a power law dependence as a distance from the critical cohesion with the

$$E_k^a(Y_k) \sim (Y_k^c - Y_k)^{\beta} \qquad \text{if} \quad Y_k < Y_k^c$$

formula and with the exponent $\beta = 1.62 \pm 0.05$, if the critical cohesion is $Y_k^c = 2.21 \cdot 10^{-4}$ (Fig. 8.) This behaviour implies that the transition shows analogies with continuous phase transitions.

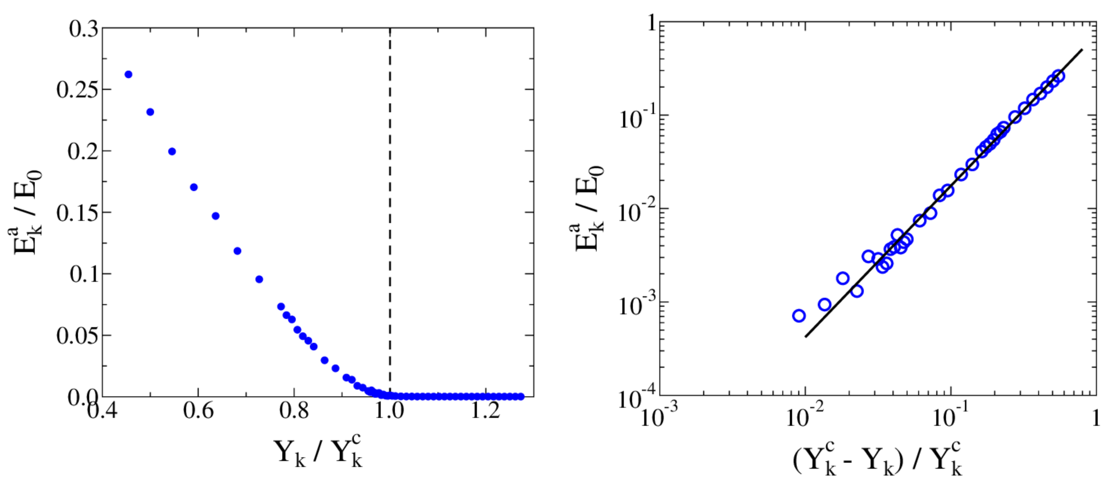


Fig. 8. The remaining kinetic energy of the system at different cohesion strength shows a power law dependence on the distance from the critical cohesion.

We found the same type of behaviour while analysing the flattening of the column. In the landslide phase the flattening ends after a period of time, so the asymptotic radius r_{max}^a has a well-defined value in this phase, but during the debris flow phase the fragments run out indefinitely due to the neglected friction. As a result, while we increase the strength of cohesion, the r_{max}^a tends to infinity with a power law dependence on the distance from the critical cohesion

$$r_{max}^a(Y_k) \sim (Y_k - Y_k^c)^{-\gamma} \qquad \text{if} \quad Y_k > Y_k^c$$

with the exponent $\beta = 0.266 \pm 0.035$. Here the critical cohesion is found to be $Y_k^c = 2 \cdot 10^{-4}$. This behaviour is illustrated in Fig. 9. before and after fitting. The result confirms our conclusion that the transition is analogous to continuous phase transitions.

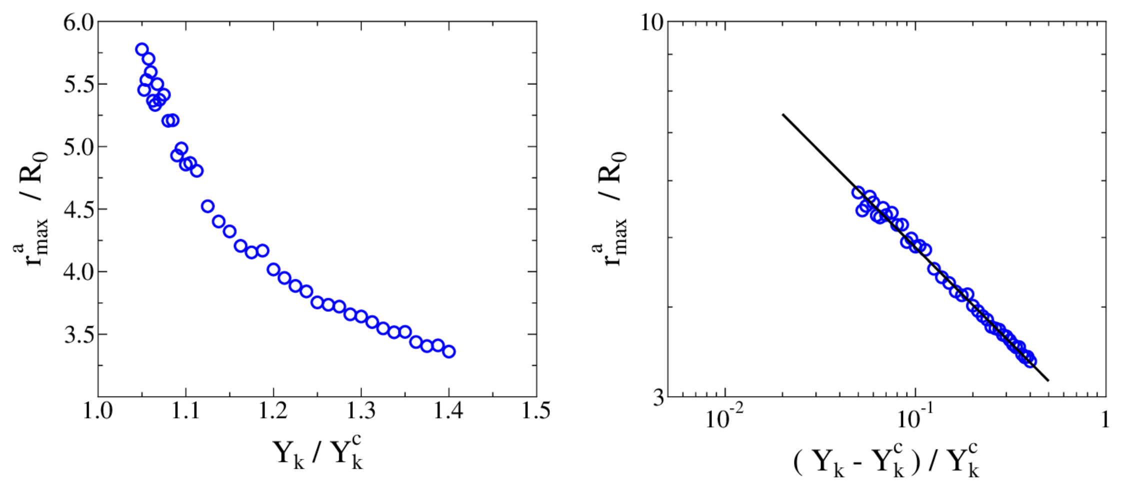


Fig. 9. The flattening of the column at different cohesion strength (in the landslide phase) shows a power law dependence on the distance from the critical cohesion.

The time evolution of the geometric profile

To examine how the collapse evolves we analysed the column's geometric profile in different moments of time during it's evolution. Due to momentum conservation the column always stays circularly symmetric, therefore, the spatial distribution of particles can be used to describe the geometric profiles. This function $h(r)$ is shown in Fig. 10. at different times of the collapse. The functional form of the evolving profiles is robust so that a scaling collapse analysis could be performed. It was found that in the early and late stages of the collapse the $h(r, r_{max})$ function has the following structure

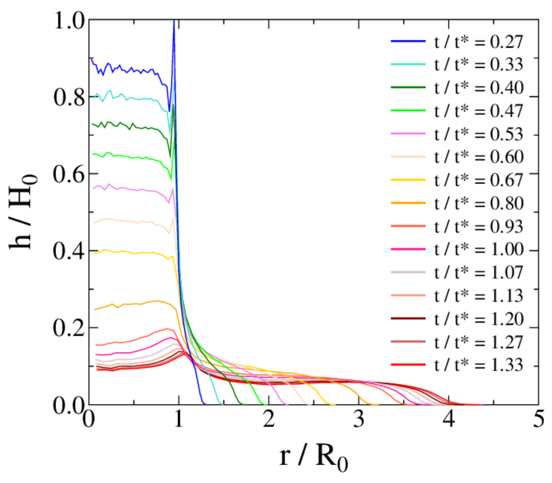


Fig. 10. The $h(r)$ function during different times of the collapse with a constant value of Y_k.

$$h(r, r_{max}(t)) = r_{max}^{-\nu} \cdot \Phi(r / r_{max}(t))$$

where $\Phi(x)$ describes the shape of the profile (Fig. 11.). The most important result is that the height h of the sample decreases as a power law of r_{max} with different exponents in the early and in the late stages.

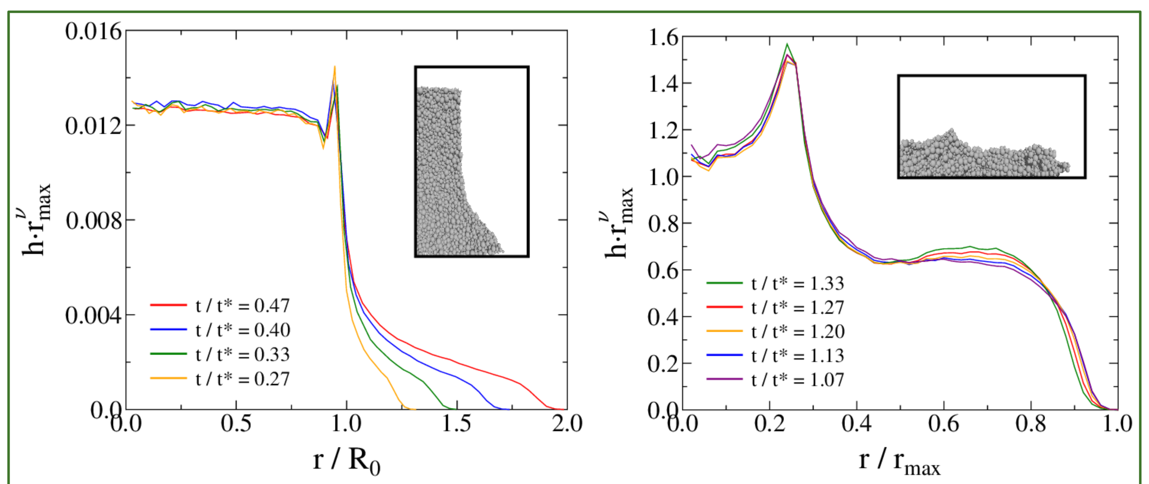


Fig. 11. The geometric profiles after scaling with different exponents in the early (left) and in the late (right) stages. $(\nu_{early} = 0.72, \ \nu_{late} = 2)$

Mass distribution of fragments

In the debris flow phase, the collapsing column breaks up into a large number of pieces. This fragmentation process is not instantaneous, instead after the fragments are formed they undergo secondary breakup as they move outward at a high speed. In order to characterize the evolution of the fragmentation process we determined the probability distribution $p(m)$ of the mass m of fragments at different times. It can be observed in Fig. 12. that at early stages the system is composed of one or two very large fragments and a few significantly smaller ones, which have a steep distribution. As time elapses, large pieces break up and a continuous distribution is obtained. A robust power law behaviour is obtained

$$p(m) \sim m^{-\tau},$$

where the value of the exponent is $\tau = 1.45$, which falls close to the corresponding value of brittle fragmentation in two dimensions [3].

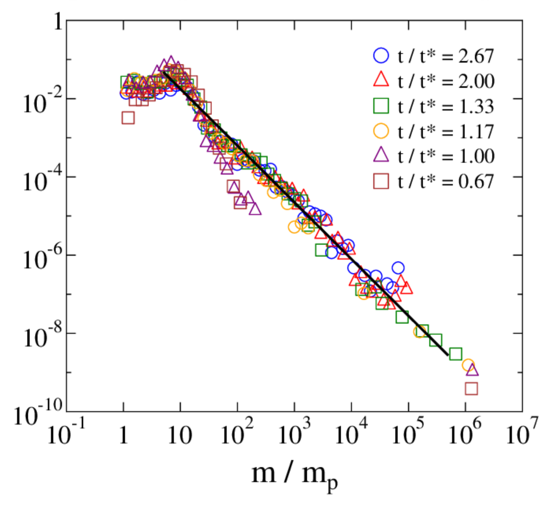


Fig. 12. Fragment mass distributions obtained at different times t. The parameter t^* is a characteristic time of the system, i.e. the time when the collapsing layer attains the final thickness.

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Contact

Csanád Szuszik (email: csanad.szuszik@gmail.com)
Ferenc Kun (email: ferenc.kun@science.unideb.hu)
Department of Theoretical Physics
University of Debrecen

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