Ising universality in the two-dimensional Blume-Capel model with quenched random crystal field

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Abstract

Using high-precision Monte-Carlo simulations based on a parallel version of the Wang-Landau algorithm and finite-size scaling techniques we study the effect of quenched disorder in the crystal-field coupling of the two-dimensional (2D) Blume-Capel model on the square lattice. We mainly focus on the part of the phase diagram where the pure model undergoes a continuous transition, known to fall into the universality class of the pure Ising ferromagnet. A dedicated scaling analysis reveals concrete evidence in favor of the strong universality hypothesis with the presence of additional logarithmic corrections in the scaling of the specific heat. Our results are in agreement with an early real-space renormalization-group study of the model as well as a very recent numerical work where quenched randomness was introduced in the energy exchange coupling. Finally, by properly fine tuning the control parameters of the randomness distribution we also qualitatively investigate the part of the phase diagram where the pure model undergoes a first-order phase transition. For this region, preliminary evidence indicate a smoothening of the transition to second-order with the presence of strong scaling corrections.

Introduction

The spin-1 Blume-Capel model [1] is defined from the Hamiltonian

$$\mathcal{H} = -J\sum_{\langle xy\rangle}\sigma_x\sigma_y + \sum_x \Delta_x\sigma_x^2, \tag{1}$$

where $J > 0$ denotes the ferromagnetic exchange interaction coupling, the spin variables $\sigma_x \in \{-1,0,+1\}$ live on a square lattice with periodic boundaries and $\langle xy\rangle$ indicates summation over nearest neighbors. Δ_x represents the crystal-field strength and controls the density of vacancies ($\sigma_x = 0$). Following Ref. [2] and the experimental motivation [3], we choose a site-dependent bimodal crystal-field probability distribution of the form

$$\mathcal{P}(\Delta_x) = p\delta(\Delta_x + \Delta) + (1-p)\delta(\Delta_x - \Delta), \tag{2}$$

where $p \in (0,1)$ is the control parameter of the disorder distribution. Note the following:

- For $\Delta = \infty$ the model is equivalent to the random site spin-$1/2$ Ising model, where sites are present or absent with probability p or $1-p$, respectively.
- For $p = 0$ the pure Blume-Capel model is recovered, see Fig. 1: For small Δ there is a line of continuous transitions (in the Ising universality class) that crosses the $\Delta = 0$ axis at $T_0 \approx 1.693$ [4, 5]. For large Δ the transition becomes discontinuous and it meets the $T = 0$ line at $\Delta_0 = zJ/2$, where $z = 4$ is the coordination number. The two line segments meet in a tricritical point at $(\Delta_t \approx 1.966, T_t \approx 0.608)$ [6, 7].
- With the inclusion of disorder ($p > 0$) it is expected that the value of Δ_0 will increase.

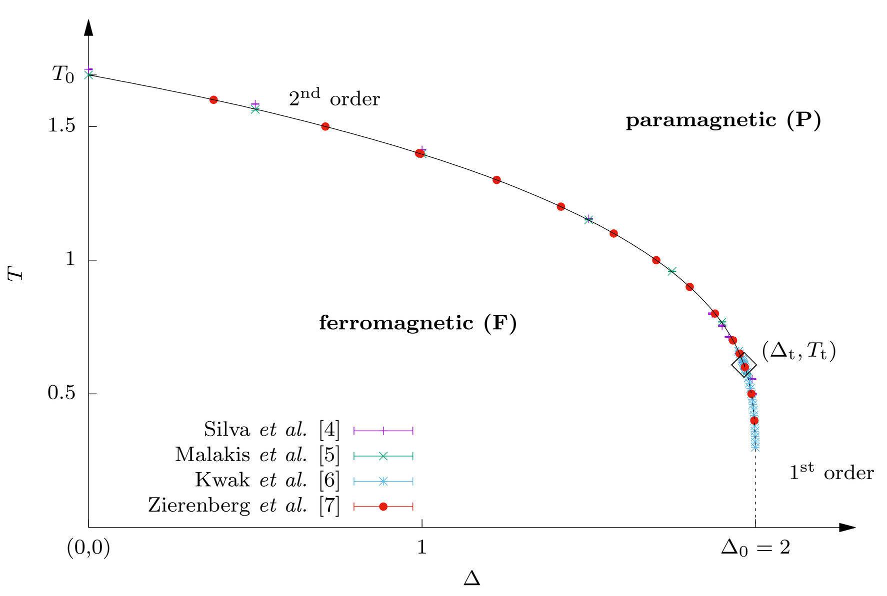

Figure 1: Phase diagram of the pure ($p = 0$) 2D Blume-Capel model in the $\Delta - T$ plane.

Simulation Details

- We used a distributed memory implementation of the Wang-Landau algorithm [8].
- We simulated the model at three values of the crystal-field coupling, $\Delta = 0.5$, 1, and $\Delta = 2$, fixing the control parameter at $p = 0.5$.
- For each value of Δ we considered linear sizes $L \in \{6, 8, 12, 16, 24, 32, 48, 64, 96\}$ and for each pair (L, Δ) we averaged over 500 random realizations.
- For $\Delta = 2$ we varied the parameter p in the regime $0 < p \leq 0.1$ to probe the ex-first-order transition regime of the phase diagram (see Fig. 4).

Results

An overview of the model's critical behavior for $p = 0.5$ is given in Figs. 2 and 3.

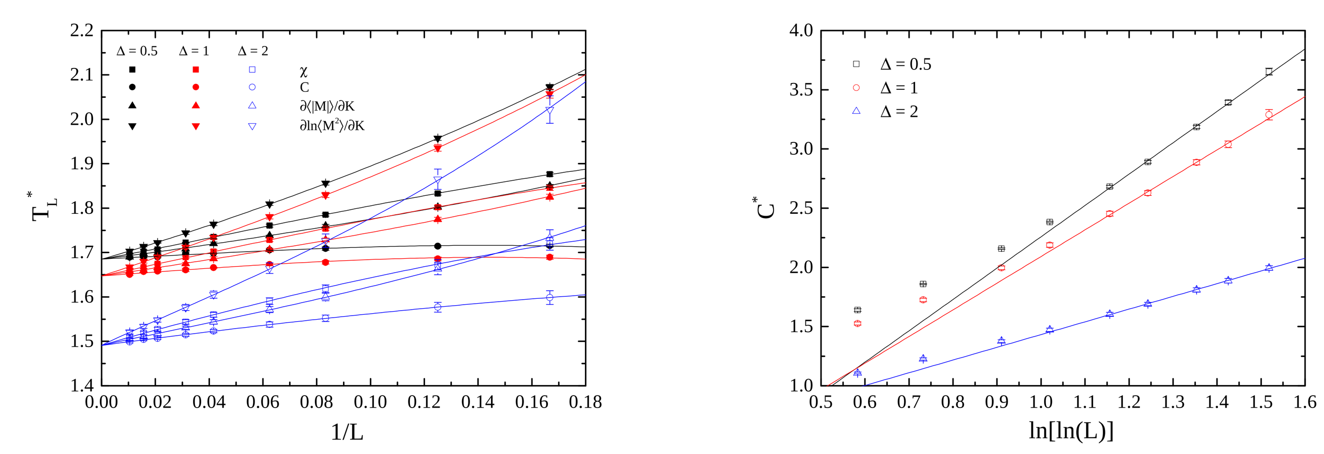

Figure 2: **Left column**: Shift behavior of several pseudo-critical temperatures according to $T_L^* = T_c + bL^{-1/\nu}(1 + b'L^{-\omega})$, where the corrections to scaling exponent is set to the expected value $\omega = 1.75$. **Right column**: Finite-size scaling behavior of the specific-heat maxima. The lines are fits of the form $C^* \sim \ln[\ln(L)]$.

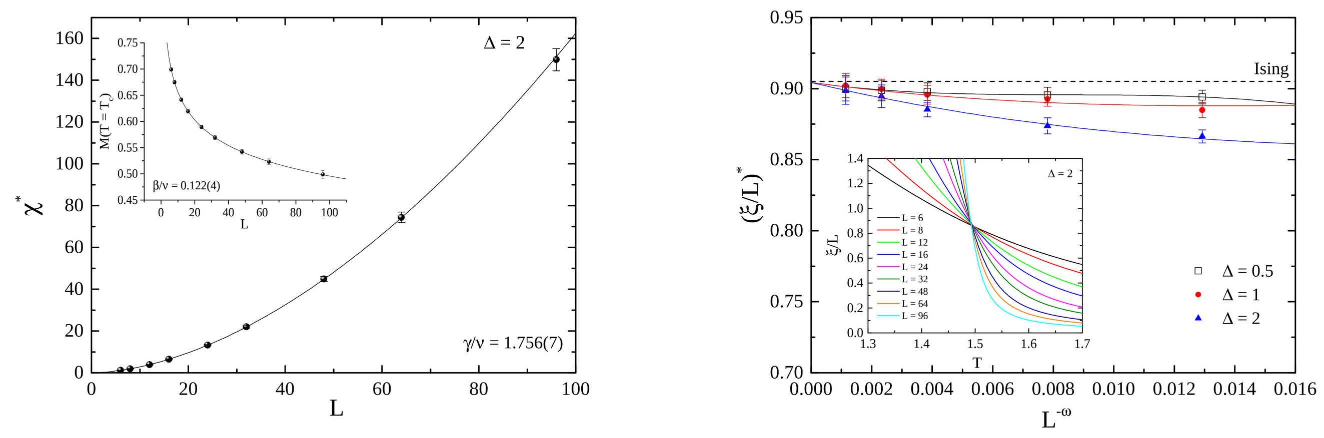

Figure 3: **Left column**: Estimation of the magnetic exponent ratios γ/ν (main panel) and β/ν (inset) for the case $\Delta = 2$. Similar results have been obtained for $\Delta = 0.5$ and 1, as well. **Right column**: Finite-size scaling of the correlation-length ratios at their crossing points $(\xi/L)^*$ of pairs of sizes $(L,\ 2L)$ (see inset). The solid lines show joint polynomial fits of third order in $L^{-\omega}$ with a common extrapolation. Note that $\omega = 1.75$ and the dashed line marks the Ising value.

Several comments are in order:

- The results for the critical temperatures are as follows: $T_c(\Delta = 0.5) = 1.6854(9)$, $T_c(\Delta = 1) = 1.6473(7)$, and $T_c(\Delta = 2) = 1.4907(6)$. The critical exponent ν was estimated to be $\nu(\Delta = 0.5) = 0.95(6)$, $\nu(\Delta = 1) = 0.99(4)$, and $\nu(\Delta = 2) = 1.04(5)$.
- For the values $\Delta = 0.5$ and 1 we observe only a slight decrease in the critical temperature with increasing Δ and only for $\Delta = 2$ a downward trend of the critical temperatures starts to settle in.
- The values for the critical temperatures of the disordered model appear to be higher that those of the pure model, especially for the case $\Delta = 2$, where the critical temperature rises from $T_c = 0 \to 1.4907$. A simple argument supporting this observed increase in the critical temperature is as follows: The case $p = 0$ corresponds to the pure model for which all crystal fields are $+\Delta$, whereas the $p = 0.5$ case brings to the model $-\Delta$ crystal fields which favor the ± 1 states.
- Our estimates for the critical exponents fall within error bars into the 2D Ising universality class.

In the final part of our work we try to elucidate the effect of disorder on the first-order transition regime of the pure model which can be addressed in transitions occurring in the small p-limit.

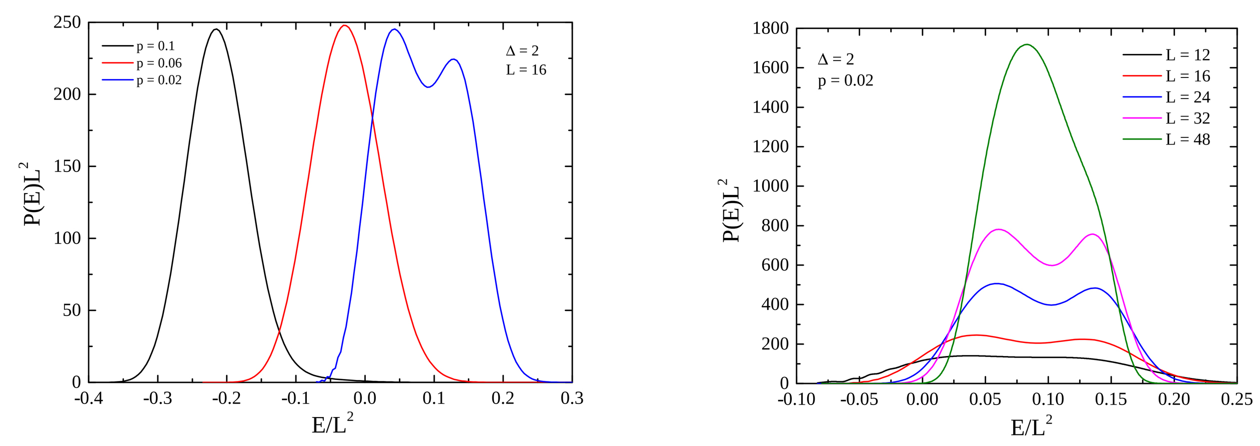

Figure 4: Typical energy probability density functions at the small p-limit. The randomness distribution (2) for $p = 0.02$ changes the pure first-order phase transition at $\Delta = 2$ into a disorder-induced continuous one, yet, with a crossover behavior for small system sizes. This crossover length appears to be of the order of $L^* \approx 48$.

Summary – Phase diagram

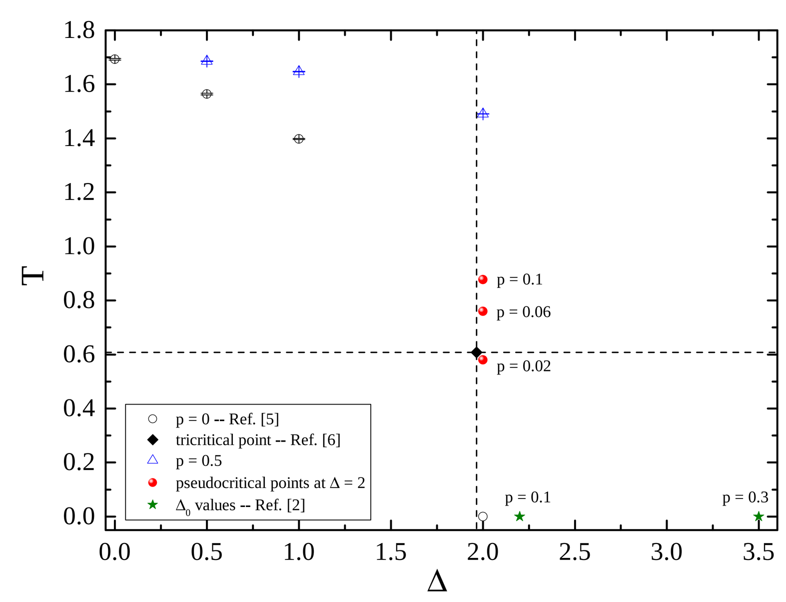

Figure 5: Selected critical and pseudocritical points of the pure and disordered Blume-Capel model [9].

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