Using high-precision Monte-Carlo simulations based on a parallel version of the Wang-Landau algorithm and finite-size scaling techniques we study the effect of quenched disorder in the two-dimensional (2D) Blume-Capel model on the square lattice. We mainly focus on the part of the phase diagram where the pure model undergoes a continuous transition, known to fall into the universality class of the pure Ising ferromagnet. A dedicated scaling analysis reveals concrete evidence in favor of the strong universality hypothesis with the presence of additional logarithmic corrections in the scaling of the specific heat. Our results are in agreement with an early real-space renormalization-group study of the model as well as a very recent numerical work where quenched randomness was introduced in the energy exchange coupling. Finally, by properly tuning the control parameters of the randomness distribution we also qualitatively investigate part of the phase diagram where the pure model undergoes a first-order phase transition. For this region, preliminary evidence indicates a smoothing of the transition to second-order with the presence of strong scaling corrections.

\section*{Introduction}

The spin-1 Blume-Capel model \cite{6} is defined from the Hamiltonian

\begin{equation}
H = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j - \sum_{x} \Delta \sigma_{x}^{2},
\end{equation}

where $J > 0$ denotes the ferromagnetic exchange interaction coupling, the spin variables $\sigma_i \in \{-1,0,1\}$ live on a square lattice with periodic boundaries and $\Delta < 0$ indicates summation over nearest neighbors. $\Delta$ represents the field-strength which controls the density of vacancies ($\sigma_x = 0$).

Following Ref. \cite{2} and the experimental motivation \cite{3}, we choose a site-dependent bimodal crystal-field

\begin{equation}
P(\Delta_{x}) = p(\Delta_{x} = \Delta_{1}) + (1-p) \delta(\Delta_{x} - \Delta_{2}),
\end{equation}

where $p \in (0,1)$ is the control parameter of the disorder distribution. Note the following:

- For $\Delta = \infty$ the model is equivalent to the random site-spin-1/2 Ising model, where sites are present or absent with probability $p$ or $1-p$, respectively.
- For $p = 0$ the pure Blume-Capel model is recovered, see Fig. 1. For small $\Delta$ there is a line of continuous transitions (the Ising universality class) that crosses the $\Delta = 0$ axis at $T$ \approx 1.630 \cite{4,5}.
- For large $\Delta$ the transition becomes discontinuous and it meets the $T = 0$ line at $\Delta = 2\beta/L$.

With the inclusion of disorder ($p > 0$) it is expected that the value of $\Delta_{c}$ will increase.

\section*{Results}

An overview of the model’s critical behavior for $p = 0.5$ is given in Figs. 2 and 3.

\section*{Summary – Phase diagram}

Several comments are in order:

- The results for the critical temperatures are as follows: $T_{c}(\Delta) = 1.635(9)$, $T_{c}(\Delta = 1) \approx 1.647(7)$, and $T_{c}(\Delta = 2) \approx 1.497(6)$. The critical exponent $\nu$ was estimated to be $\nu(\Delta = 0) = 0.95(6)$, $\nu(\Delta = 1) = 0.99(4)$, and $\nu(\Delta = 2) = 1.05(4)$.
- For the values $\Delta = 0.5$ and $1$ we observe only a slight decrease in the critical temperature with increasing $\Delta$ and only for $\Delta = 2$ a downward trend of the critical temperatures start to settle in.
- The values for the critical temperatures of the disordered model appear to be higher that those of the pure model, especially for the case $\Delta = 2$, where the critical temperature rises from $T_{c} = 0.1 \rightarrow 1.497$. A simple argument supporting this observed increase in the critical temperature is as follows: the case $p = 0$ corresponds to the pure model for which all crystal fields are $\Delta = 0$, whereas the $p = 0.5$ case brings to the model $\Delta = \Delta_{c}$ crystal fields which favor the $\pm 1$ states.
- Our estimates for the critical exponents fall within error bars into the 2D Ising universality class.

In the final part of our work we try to elucidate the effect of disorder on the first-order transition regime of the pure model which can be addressed in transitions occurring in the small $\Delta$-limit.

\section*{References}

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