Hidden Markov Models: A Breakdown of Random Matrix Universality & The Brain Criticality Hypothesis

Faheem Mosam & Dr. Eric DeGiuli
Ryerson University

INTRODUCTION AND BACKGROUND

Biological systems need to react to stimuli over a broad spectrum of timescales, how these timescales can emerge without external fine-tuning is still a puzzle. We consider discrete Markovian systems which are governed by transition matrices, allowing us to leverage results from random matrix theory. An ensemble of transition matrices is considered, and we introduce and motivate temperature-like parameters that control the dynamic range of matrix elements. We show that a phase transition from full to sparse occurs whereby random matrix theory results breakdown. This phase transition signifies the emergence of a nontrivial Shannon entropy and is accompanied by a peak in complexity as measured by predictive information. The results are then applied to fMRI data of human subjects at wakeful rest and show that brain activity lies close to the phase transition when engaged in unconstrained, task-free cognition.

Hidden Markov Models (HMM)

A HMM relates a set containing N hidden states \( \{ s_i \} \) with a set containing T observable states \( \{ a_t \} \). This is done via a transition matrix \( M \) of size \( N \times N \) with elements \( M_{ij} = P(s_{t+1} = i | s_t = j) \). Naturally, the elements of \( M \) are normalized such that \( \sum_j M_{ij} = 1 \) and \( m_{ii} \geq 0 \). The cut degree is the number of transitions to nodes out of a given node.

Hypothetical and Specific Aims/Objectives

Model systems in a way that allows identification of the phase transition from noisy behaviour into an ordered phase. Moreover, show that this phase transition identifies a point of criticality in dynamic HMM systems.

MATLAB AND METHODS

All code for this project is written in MATLAB, split between two main sections: one for the random HMM (RHMM) and one for the human data analysis (NHMM). The RHMM code is adapted from a previous project based off of DeGiuli(2019) [5]. The NHMM code was written from scratch with the exception of the generation of plots which used sections from the RHMM code.

The RHMM code consisted of two sets of data, each with 5 different sized matrices and 10 different temperature parameters. The first set had 300 replicas per matrix size and temperature with each replica being sampled 100 times with sequences of maximum length 4000. The second set of data had 1000 replicas per matrix size and no sampling occurred.

The NHMM code was developed by Vidaurre [1]. It consisted of 820 subjects which each sat for four sessions for 1200 time steps each. The data was processed accordingly such that transition matrices could be built and measured.

HYPOTHESIS AND SPECIFIC AIMS/OBJECTIVES

The predicted transition represents a point of criticality for dynamical HMM systems, by using this first principles theoretical prediction we show a simple scenario for the emergence of long time scales in discrete Markovian systems, by varying the dynamic range of matrix elements. The results are then used to test the brain criticality hypothesis using RHMM data.

DISCUSSION AND CONCLUSIONS

• Figure 4 shows the Shannon entropy rate of RHMM data normalized by its maximum value log(N) plotted against the temperature parameter \( \epsilon \), which is normalized by its critical value. The normalized entropy rate tends to unity for large temperature, indicating that the transition sequences are indistinguishable from random noise, while low temperatures are nearly equal to 0, indicating that sequences are nearly deterministic.

• Figure 5 shows the probability distribution of the normalized Shannon entropy rate becomes \( H/\log(N) \) at large temperatures, which each sat for four sessions for 1200 time steps each. The data was processed accordingly such that transition matrices could be built and measured.

REFERENCES


ACKNOWLEDGEMENTS

EDG acknowledges support of the Natural Sciences and Engineering Research Council of Canada (NSERC), Discovery Grant RGPIN-2020-04762.