Effects of inhomogeneous bulk magnetization of magnetic particles on the magnetic energy of a magneto-sensitive elastomer.

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Introduction

Magneto-sensitive elastomers are rubber-like materials, elastic properties of which are highly sensitive to the external magnetic field. They belong to the broader class of composite materials and consist of two subsystems: elastic (which usually is described with Hooke's law or with particular nonlinear model) and magnetic (which is usually described with point-like dipole-dipole interactions). However, when distance between the particles become comparable with their size, effects of inhomogeneous bulk magnetization become increasingly important. Ignorance of such effects may lead to inaccuracies in the theoretical predictions of the mechanical properties of the material. Here we concentrate our attention on magnetic subsystem and inhomogeneous bulk magnetization. We consider non-magnetic elastic subsystem filled with equal micron size magnetically soft spherical particles.

Magnetic energy of two particles

In order to find the magnetic energy of the two spheres in the non-magnetic medium ($\mu=\mu_{\text{medium}}/\mu_0=1$) without electrical currents, one need to solve the Laplace equation (LE). The result of this computation, which was done elsewhere, can be presented as a series in terms of dimensionless inverse distance as follows

$$U_{\text{mag}}=-3\mu_0\beta_1V_pH_0^2\sum_{n=0}^{\infty}w_n\left(\frac{a}{r}\right)^n=2U_0\sum_{n=0}^{\infty}w_n\left(\frac{a}{r}\right)^n, \tag{1}$$

where every coefficient ω_k is of a type $\omega_k=C_k^{(0)}+C_k^{(2)}\cos^2\theta$ and $\beta_n=\frac{n(\mu_p-1)}{(n\mu_p+n+1)}\sim 1$. To compare this result and to better understand it from more "physical" point of view, we develop the model of self-consistent dipole (SCD). We consider spherical particles mutually magnetizing each other via dipole interaction.

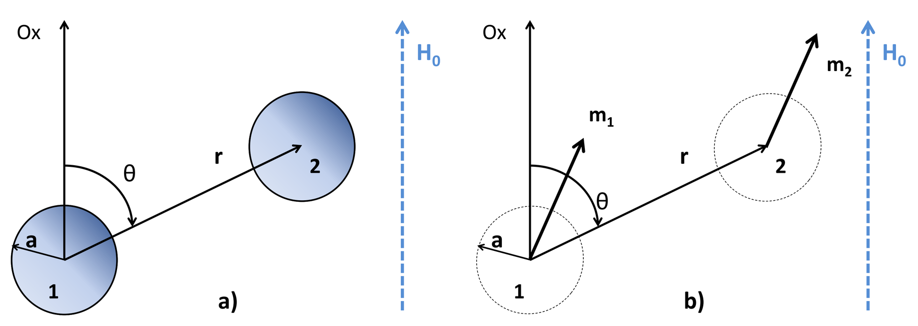

Fig. 1: a) Schematic depiction of inhomogeneously magnetized spheres; b) their map into self-consistent dipoles.

We wright the equation as follows

$$\mathbf{m}_i=3\beta_1V_p\Big(\mathbf{H}_0+\frac{1}{4\pi}\frac{3(\mathbf{m}_{3-i}\cdot\mathbf{r})\mathbf{r}-r^2\mathbf{m}_{3-i}}{r^5}\Big). \tag{2}$$

This allows us to obtain the energy of two self-consistent dipoles

$$U_{\text{SCD}}=-3\mu_0\beta_1V_pH_0^2\frac{1+\beta_1(\frac{a}{r})^3(3\cos^2\theta-2)}{1-\beta_1(\frac{a}{r})^3-2\beta_1^2(\frac{a}{r})^6}=2U_0\sum_{n=0}^{\infty}\omega_n\left(\frac{a}{r}\right)^{3n}. \tag{3}$$

Magnetic energy of the sample and a shape factor

The energy of N particles inside the sample can be written as follows

$$U_N=N_pU_0+\frac{1}{2}\sum_{\substack{j,i\\ i\neq j}}U_{ij}^{(\text{int})}, \tag{4}$$

where we ignore higher than pairwise interactions. Using Eq. (1) we can see

$$U_{ij}^{(\text{int})}=2U_0\left[a^3\frac{3\cos^2\theta_{ij}-1}{r_{ij}^3}+a^6\frac{3\cos^2\theta_{ij}+1}{r_{ij}^6}+\cdots\right]. \tag{5}$$

This lead to the following expression of energy density

$$u=u_0\Big(1+3\phi\beta_1f^{\text{eff}}\Big);\quad f^{\text{eff}}=f^{(3)}+f^{(6)}+\cdots \tag{6}$$

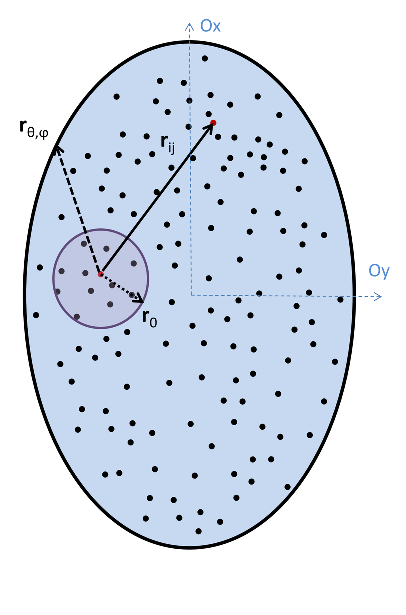

Fig. 2: Micro-sphere inside the spheroidal sample.

Now, we want to split the volume of the sample into two parts: the micro-sphere and the rest of the sample. Inside micro-sphere one should proceed with summation, and outside it is convenient to perform integration. Thus, we can split dimensionless parameter f as follows $f^{\text{eff}}=f^{\text{eff}}_{\text{micro}}+f^{\text{eff}}_{\text{macro}}$. The integral part consist of the terms of the following type

$$f_{\text{macro}}^{(k)}=C_0\Big(\frac{a}{r_0}\Big)^{k-3}+C_1\left[\left(\frac{a}{A}\right)^{k-3}+\left(\frac{a}{B}\right)^{k-3}\right]G^{(k)}(\gamma), \tag{7}$$

where $G^{(k)}(\gamma)$ some finite function of aspect ratio $\gamma=A/B$

$$G^{(k)}(\gamma)=\frac{1}{1+\gamma^{k-3}}\int_{-1}^{1}dx(a_kx^2+b_k)\Big[(1-\gamma^2)x^2+\gamma^2\Big]^{(k-3)/2}. \tag{8}$$

As a result, the macroscopic sample limit yields well known continuum mechanic result

$$f_{\text{macro}}^{\text{eff}}=\sum_k f_{\text{macro}}^{(k)}\xrightarrow[\text{macroscopic sample}]{}f_{\text{macro}}^{(3)}==\frac{1}{3}-N(\gamma), \tag{9}$$

where $N(\gamma)$ is demagnetizing factor of spheroidal sample.

Comparison of different energies

To compare energies obtained according different approximations we plot them as functions of scaleless distance r/a for four different configurations ($\theta=0°,30°,60°,90°$). Slope of each curve define the force acting on particles. In the cases $\theta=0°,30°$ one can observe that all approximations qualitatively agree, and that the forces are attractive. In the case 90° all approximations predict repulsive force. And for 60° we have discrepancies in predictions for different models.

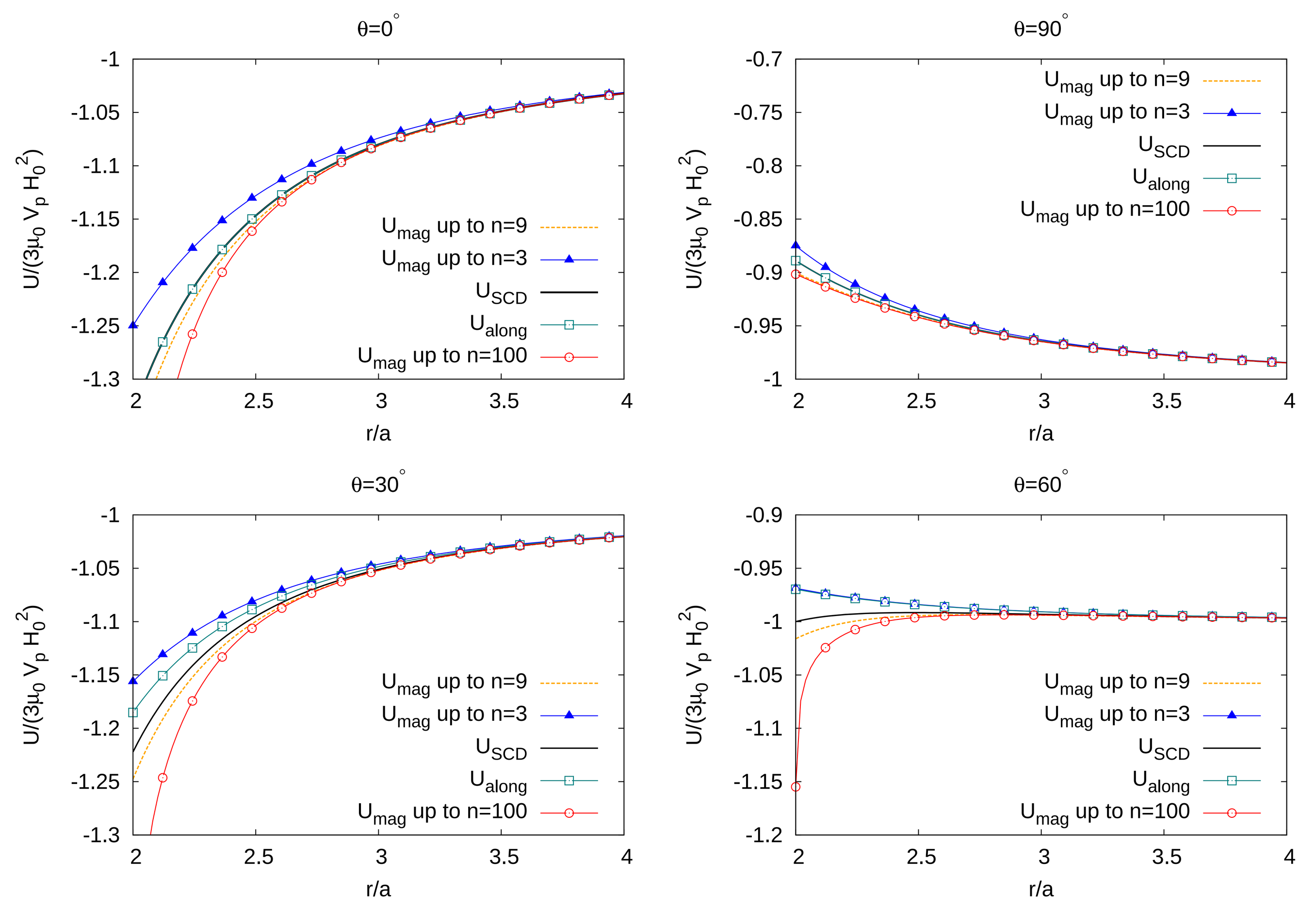

Fig. 3: Two particle energies for four different configurations.

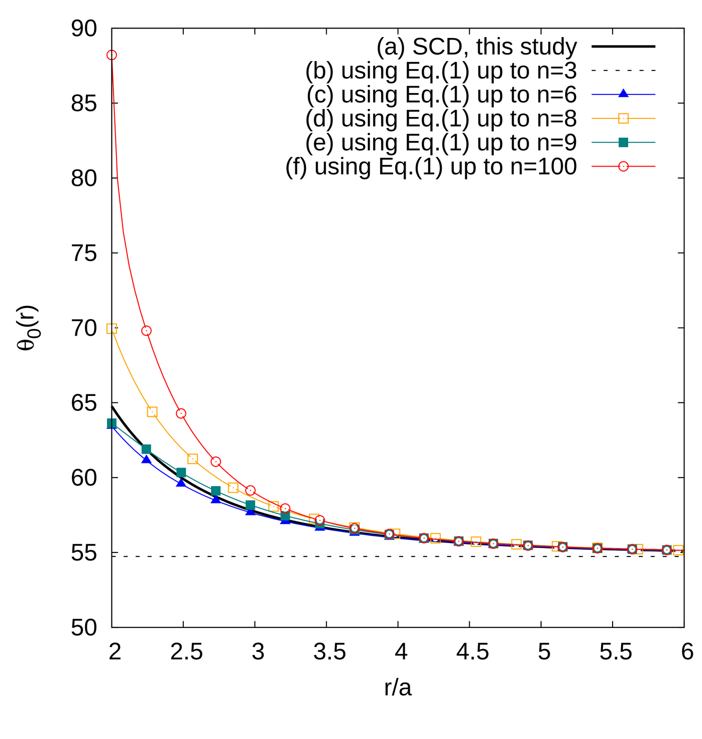

Fig. 4: Zero force angle θ_0 as function of r/a.

Now, using Eq. (1) we can write the general expression of zero force angle

$$\theta_0(r/a)=\arccos\sqrt{-\frac{F_0(r/a)}{F_2(r/a)}}. \tag{10}$$

Higher values of this function mean wider range of configurations for which attraction is predicted. In the case of inhomogeneously magnetized spheres (red), for contact distance, we can see attraction almost for any configuration. SCD model also predicts attraction for $\theta_0(2)$ up to 64.8°. Peculiarly, higher approximation (up to $n=9$) is worse than (up to $n=8$)! In addition eight power term, $(a/r)^8$, can not be obtained in SCD model at all (see Eq. (3)).

Discussion

1) It is interesting that more accurate models predict attraction for such a high θ, thus enhancing particles ability to form clusters. 2) It looks like the changes in the magnetic energy model do not lead to changes in a shape factor of the sample.

Acknowledgments

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