Controlling particle currents with evaporation and resetting from an interval

G. Tucci, A. Gambassi, S. Gupta, E. Roldan

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1 SISSA - International School for Advanced Studies
2 Department of Physics, Ramakrishna Mission Vivekananda Educational and Research Institute
3 ICTP - The Abdus Salam Centre for Theoretical Physics

Stochastic resetting from an interval

Model: one dimensional Brownian motion resetting at space dependent rate $r_c(x) = r \theta (a - |x|)$ to a prescribed resetting point $x_r$.

Master Equation: $\partial_t P(x, t | x_0) = D \partial_x^2 P(x, t | x_0) - r_c(x) P(x, t | x_0) + \delta(x - x_r) \int dy r_c(y) P(y, t | x_0)$.

Diffusion Loss due to Resetting Gain due to Resetting

i) Resetting induces a cusp at $x_r$ at all times

ii) Diffusion is responsible for Gaussian tails

Brownian yet non-Gaussian: non-Gaussian distribution but

i) $\sigma^2(t) \approx D_{\text{eff}} t$ for $t \gg 1$, with

$1 - \frac{2}{\pi} \frac{D_{\text{eff}}}{D} \leq 1$.

ii) $D_{\text{eff}}(x)$

Resetting on a ring

Model: Brownian particle resetting at space dependent rate $r_c(x)$ on $(-L, L)$ with periodic boundary conditions; this ensures the existence of a stationary state.

Particles can reset both clockwise and counterclockwise: this is fixed by a protocol.

Resetting is instantaneous

Gauge invariance

Infinite possible particle currents Only one $P(x, t | x_0)$

The total current $J$ is composed by two contributions

i) $J_{\text{diff}}$ independent of the protocol;

ii) $J_{\text{res}}$ dependent of the protocol.

Application: Modelling backtrack recovery of RNA polymerase

RNA polymerases along a DNA template during transcriptional pauses. Two recovery mechanisms from the inactive state (“backtracking”): i) Brownian diffusion, and ii) active cleavage of the backtracked RNA. Model: evolution of the backtrack depth $x \geq 0$, i) diffusion in $d = 1$ starting from $x_0 > 0$ with an absorbing boundary in the origin, ii) resetting of the backtrack depth $x \to x_r = 0$ at a rate $r$ from the region $(0, a)$.

Distribution of the length of cleaved RNA

$Q_{\text{res}}(x = ay | x_0 = ay_0)$

$= \rho \sinh(y_0 \rho) \left[ 1 - \theta (1 - y_0) \left( 1 - \cosh((1 - y_0) \rho) \right) \right]$.

Cleavage efficiency $\eta_{\text{res}}(y_0) \equiv \eta(x_0 = ay_0)$

$= 1 - \frac{\cosh((1 - y_0) \rho) + \theta (y_0 - 1) \left( 1 + \cosh((1 - y_0) \rho) \right)}{\cosh \rho}$

with $\rho = a \sqrt{r_D y_c} = \min(y, y_0)$, and $y_+ = \max(y, y_0)$.