MEDICAL UNIVERSITY OF VIENNA

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Balanced and fragmented phases in societies with homophily and social balance

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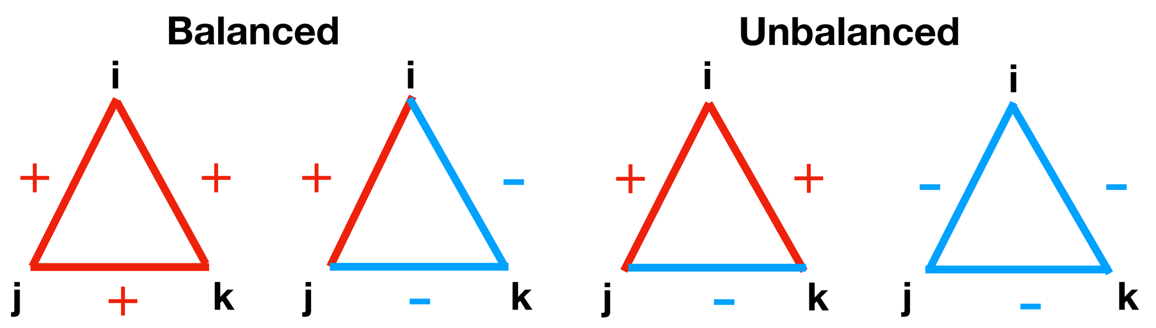
Abstract

We try to understand the recent increase of social fragmentation with a simple model where we use spins to represent G-dimensional vectors of binary opinions of individuals and use a positive (negative) link weight to represent friendship (enmity), respectively. We take into account the joint effects of (1) homophily—the tendency of people with similar opinions to establish positive relations, and (2) social balance—the tendency to establish balanced triadic relations. These two mechanisms are incorporated in a localized Hamiltonian that minimizes social stress through the co-evolution of opinions of individuals and their social networks. We show how the likelihood of social fragmentation increases as individuals care more about social balance. We identify the critical size of the social neighbourhood, Q_c, above which society must fragment into communities that are internally cohesive and hostile towards other groups.

Model



Consider a society of N individuals. Each individual i is characterized by an attribute vector, $\mathbf{A}_i = \{a_i^\ell\}$, where $a_i^\ell \in \{-1,+1\}$; $\ell \in 1,\dots,G$; i is embedded in a fixed network topology. The relation between two agents i and j is determined by: $J_{ij} = \mathrm{sign}(\mathbf{A}_i \cdot \mathbf{A}_j)$,i. e. by their distance in the attribute space. $J_{ij} = 1$ indicates friendship, $J_{ij} = -1$ enmity. Each agent i has a social stress level, $H^{(i)}$, defined as

$$H^{(i)}(\mathbf{A}) = -\frac{1}{G}\sum_j J_{ij}\mathbf{A}_i \cdot \mathbf{A}_j - \sum_{(j,k)_{Q_i}} J_{ij}J_{jk}J_{ki}\,. \qquad (1)$$

The first sum extends over all k_i neighbours of i, while the second is restricted to Q_i triads relevant to i's stress-calculation. Since Q_i are chosen at each step of the dynamics among those that i belongs to, Q_i limits the extent to which the social network can change. Specifically, those edges that do not belong to the Q_i triads, will *not* be updated.

Assuming agents try to minimize their individual social stress over time, we implement the following dynamics

1. *Initialize.* For any pair of connected agents, i and j, we set $J_{ij} = \mathrm{sign}(\mathbf{A}_i \cdot \mathbf{A}_j)$, where the opinion vector, $\mathbf{A}_i$, of each node i has components randomly chosen to be 1 or -1 with equal probability.
2. *Update.* Pick a node i randomly and choose Q of its triads randomly. Flip one of i's attributes at random. Let $\tilde{\mathbf{A}}_i$ be its new opinion vector. For each of the chosen triads, the weights of the two links adjacent to i are recomputed as $\tilde{J}_{ij} = \mathrm{sign}(\tilde{\mathbf{A}}_i \cdot \mathbf{A}_j)$. $\tilde{J}$ is the new matrix. Compute the change in stress is $\Delta H^{(i)} \equiv \tilde{\mathcal{H}} - \mathcal{H}$. Update $\mathbf{A}_i \to \tilde{\mathbf{A}}_i$ and $J_{ij} \to \tilde{J}_{ij}$ with probability, $\min\left\{e^{-\Delta H^{(i)}}, 1\right\}$, otherwise leave it unchanged.
3. Continue with the next update by returning to step 2.

Order Parameter

$$f = \frac{n_+ - n_-}{n_+ + n_-}, \qquad (2)$$

where n_+ and n_- are the number of balanced and unbalanced triangles.

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Result

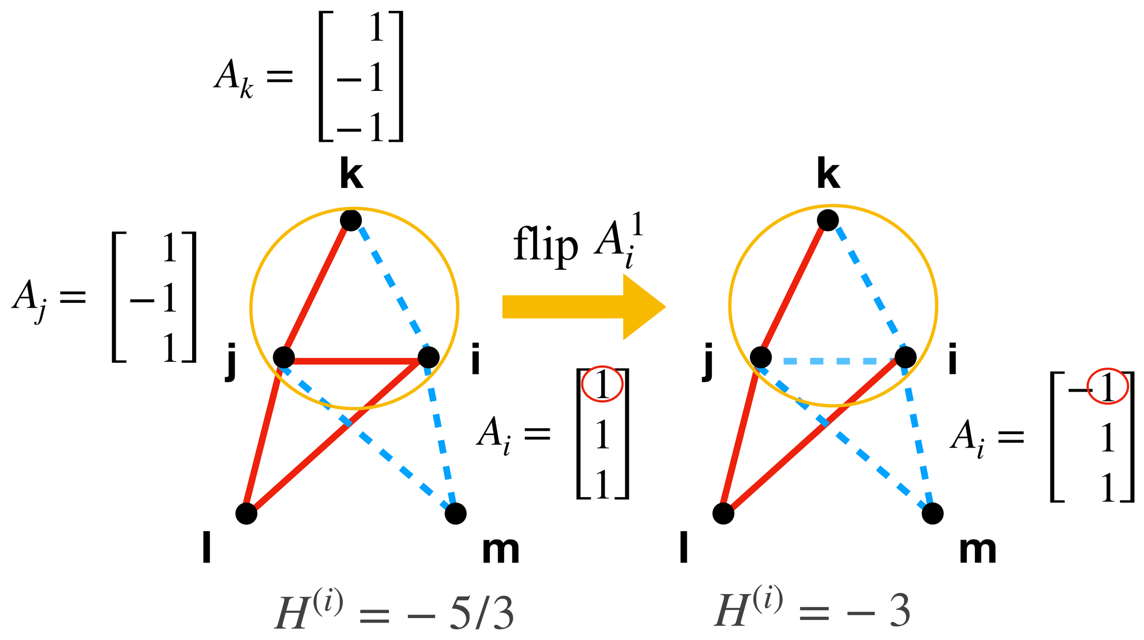

Figure 1:Co-evolutionary interplay of opinions and links. Red (blue) links denote positive (negative) relationships. The chosen triad, (i,j,k) is circled. As agent i flips one of its attributes, A_i^1, this triad becomes balanced and i decreases its individual stress from $-5/3$ to -3. The opinion vectors of l and m are $\mathbf{A}_l = \mathbf{A}_i$ and $\mathbf{A}_m = -\mathbf{A}_i$, respectively.

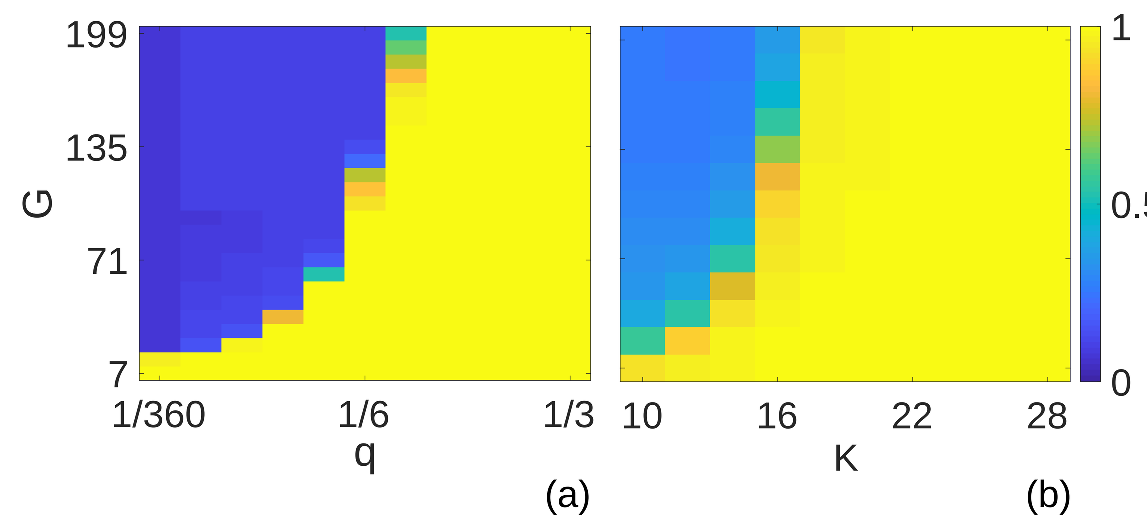

Figure 2:Order parameter, f, (a) as a function of q and G for $K = 32$, and (b) as a function of K and G with $q = 1/3$ and $N = 400$.

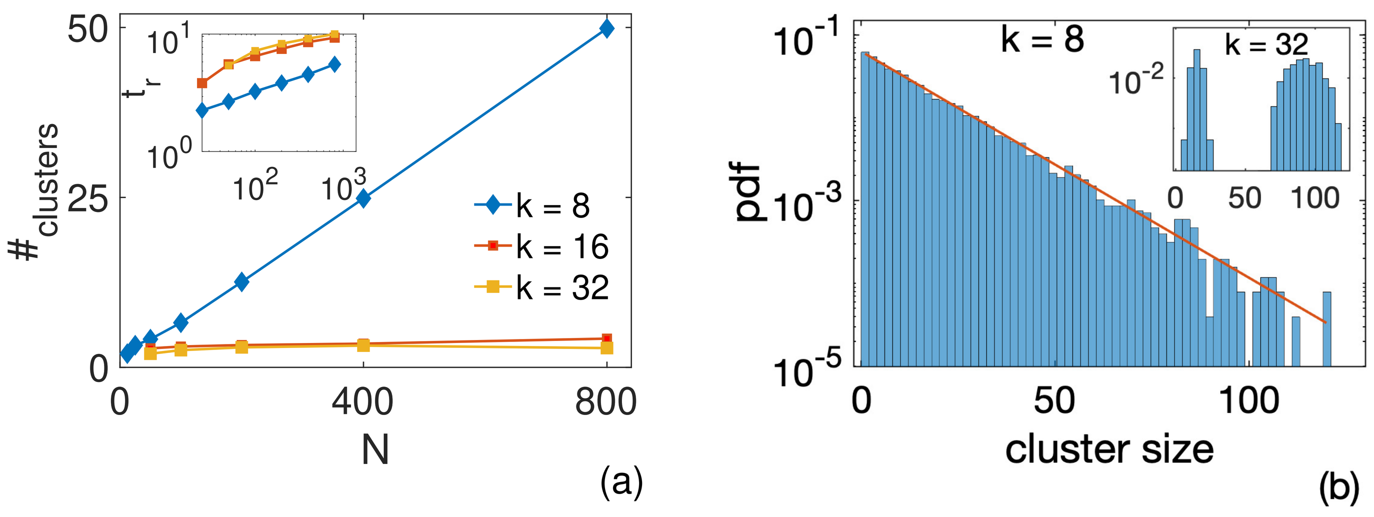

Figure 3:(a) Average number of clusters and average time to reach the balanced states, t_r, with $f = 1$ (inset), as a function of N for $K = 8, 16, 32$ with $N = 12, 25, 50, 100, 200, 400, 800$, $G = 9$ and $Q = 16$. (b) Probability density function of cluster sizes for $K = 8$ (main plot) and $K = 32$ (inset); $N = 200$, $G = 9$ and $Q = 16$.

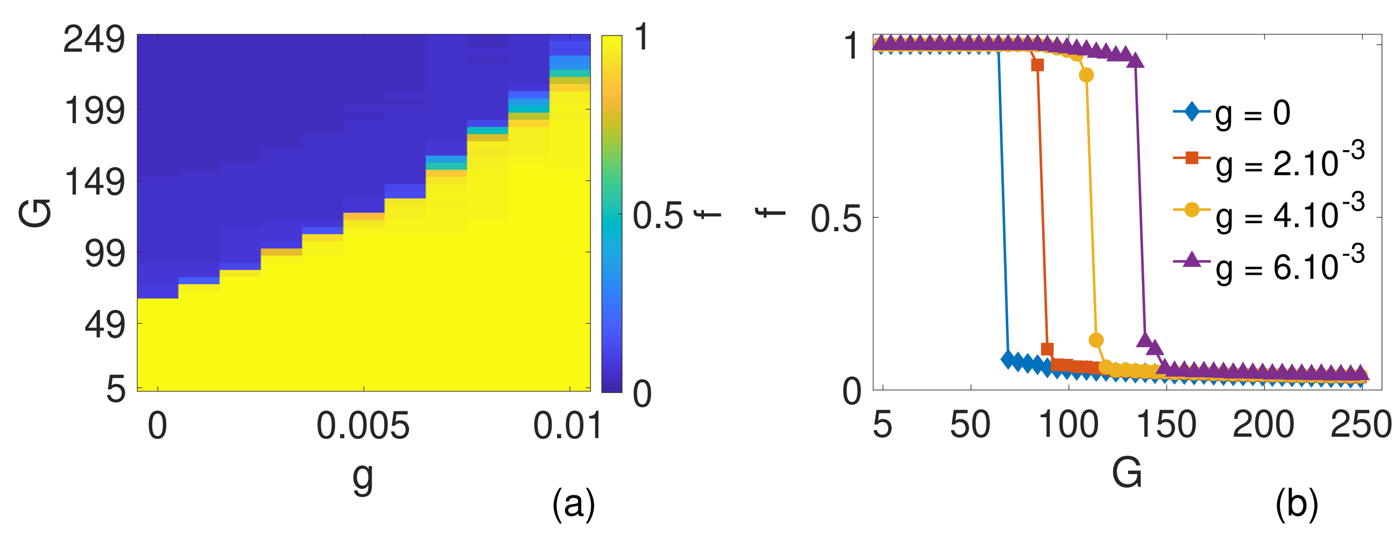

Figure 4:(a) f as a function of g and G, where g is the coupling constant of the global Hamintonian $\tilde{H} \equiv -\frac{1}{2G}\sum_{(i,j)} J_{ij}\,\mathbf{A}_i \cdot \mathbf{A}_j - g\sum_{(i,j,k)} J_{ij}J_{jk}J_{ki}$. (b) Section of the phase diagram for various g.

References

[1] T. M. Pham, A. C. Alexander, J. Korbel, R. Hanel, and S. Thurner. Balanced and fragmented phases in societies with homophily and social balance, 2020.