BELARUSIAN STATE UNIVERSITY

Analytical estimation of the percolation threshold for mixed site-bonds problems by the renormalization group method

A.S.Fedotov*, I.Zur, Y.Tsitavets and K.Stepanishin

fedotov.alehandro@gmail.com

Physics Faculty of Belarusian State Univeristy, 2 Nezalezhnasci av., Minsk

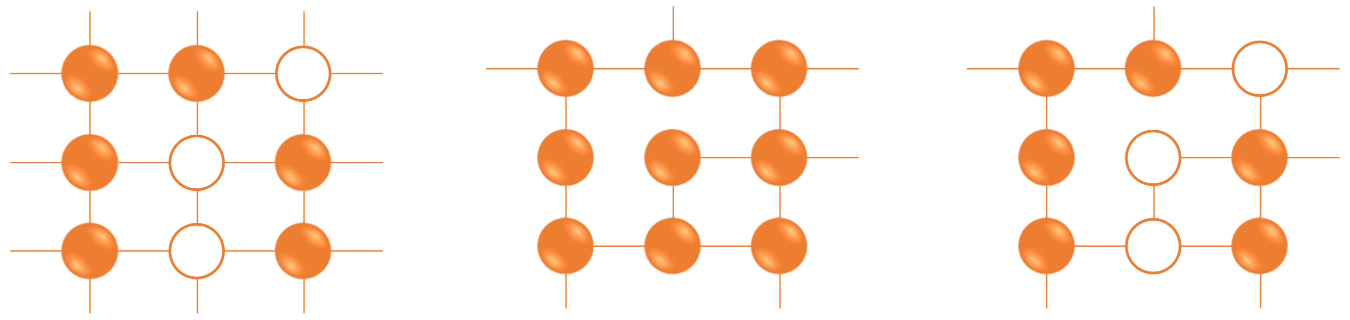
MECO46
The 46th International Conference of the Middle European Cooperation in Statistical Physics
May 11-13, 2021
Riga, Latvia

Motivation

Many properties of composite materials such as an electrical conduction, dielectric response, diffusion, etc. are closely related to the geometrical arrangement of the constitutive phases. Percolation theory, whose objective is to characterize the connectivity properties in random geometries, provides a frame for the theoretical description of random media.

Mixed percolation theory is applicable in studies of ring vaccination, road traffic, capillary phenomena in porous media, pathogen mutations, and others. In recent years, great progress has been made in the field of numerical methods of percolation theory; however, analytical descriptions for many important cases still remain open and of interest.

Lattice percolation problems



Site | Bond | Mixed (we study this one)

Main question

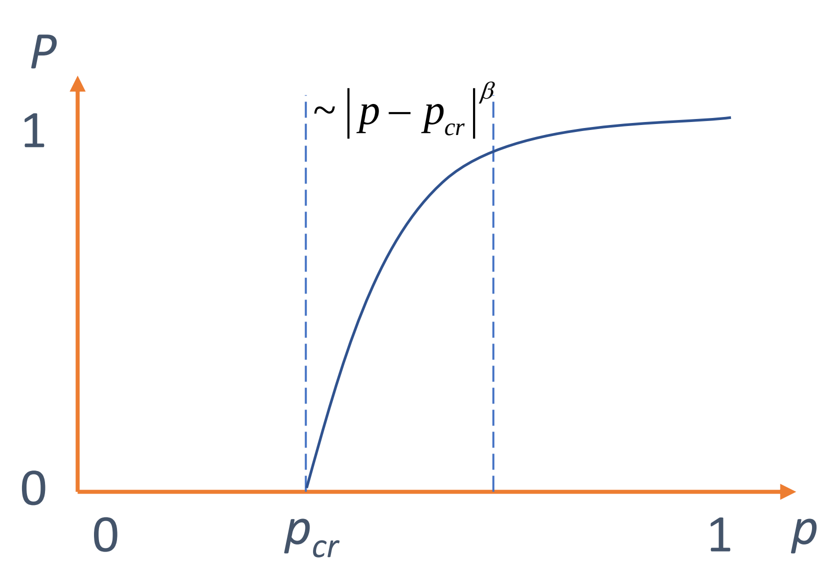
What is the maximum fraction of sites/bonds/both which can be removed while infinite cluster still exists?

Percolation probability (function) $P(p)$ that infinite cluster exists:

p – probability that individual site (bond) is conductive;

S_∞ – size of the largest cluster in system;

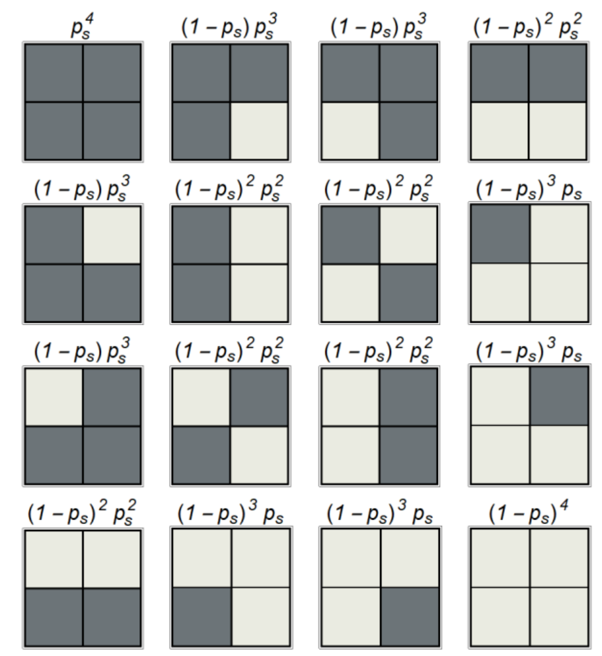
S_Ω – summed sizes of all clusters.



Critical fraction of conductive sites when cluster appears

Classical renormalization group method

The method consists in sequential replacement of groups of nodes (elementary cells) with single nodes. The operation is iteratively repeated until the entire computational domain turns into a single conductive or non-conductive node.



Probabilities of different configurations of **2x2 cell**

Probability of site to be conductive on i+1 iteration:

$$p_s^{i+1} = \sum_s config_s = \left(p_s^i\right)^2\left(2-\left(p_s^i\right)^2\right)$$

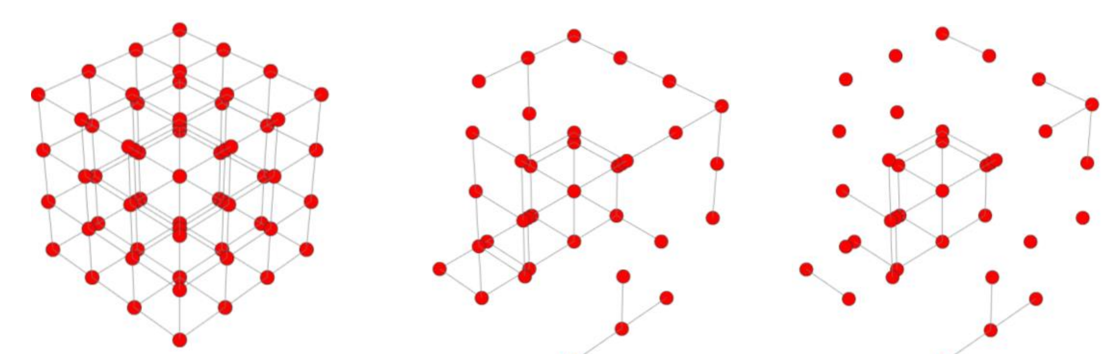
Fixed mapping points indicate the position of the percolation threshold:

$$p_s = \left(p_s\right)^2\left(2-\left(p_s\right)^2\right)$$

Real root is percolation threshold p_{cr} = 0.61 for site problem

(numerical value is 0.59 for square lattice)

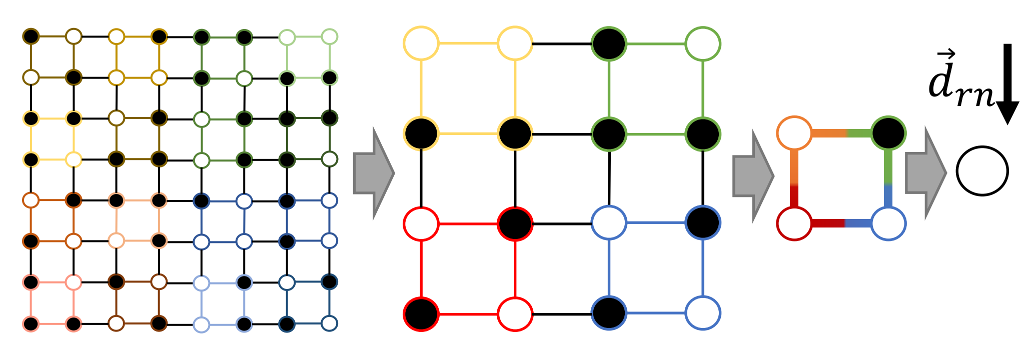
Numerical method I



Take perfect lattice | Remove some sites | Remove some edges

The result is a graph which can be analyzed by breadth-first search

Numerical method II



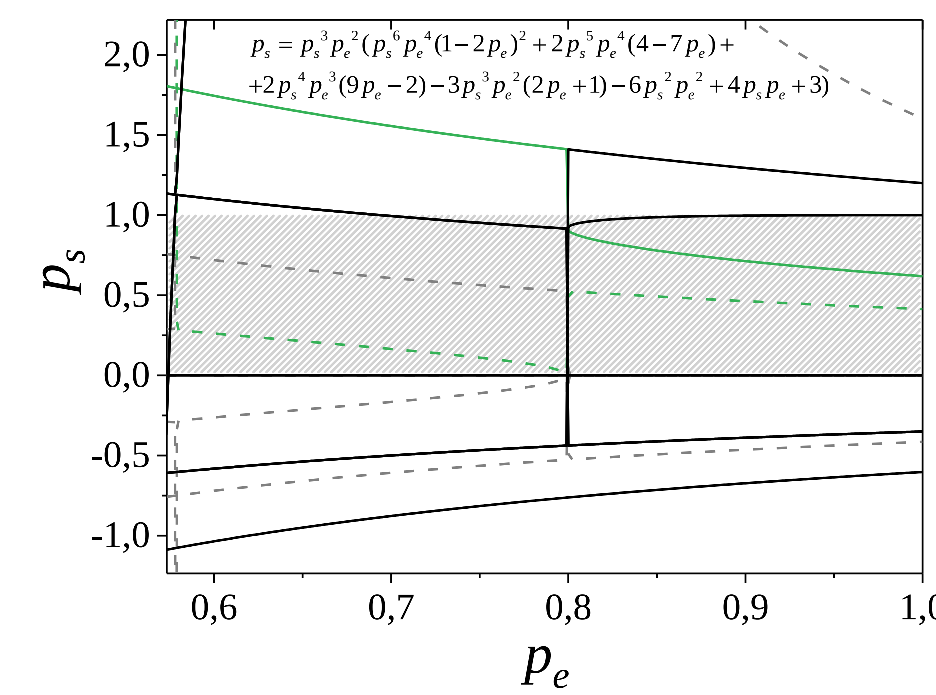
Successively replace elementary cell by single site

The result is a single site

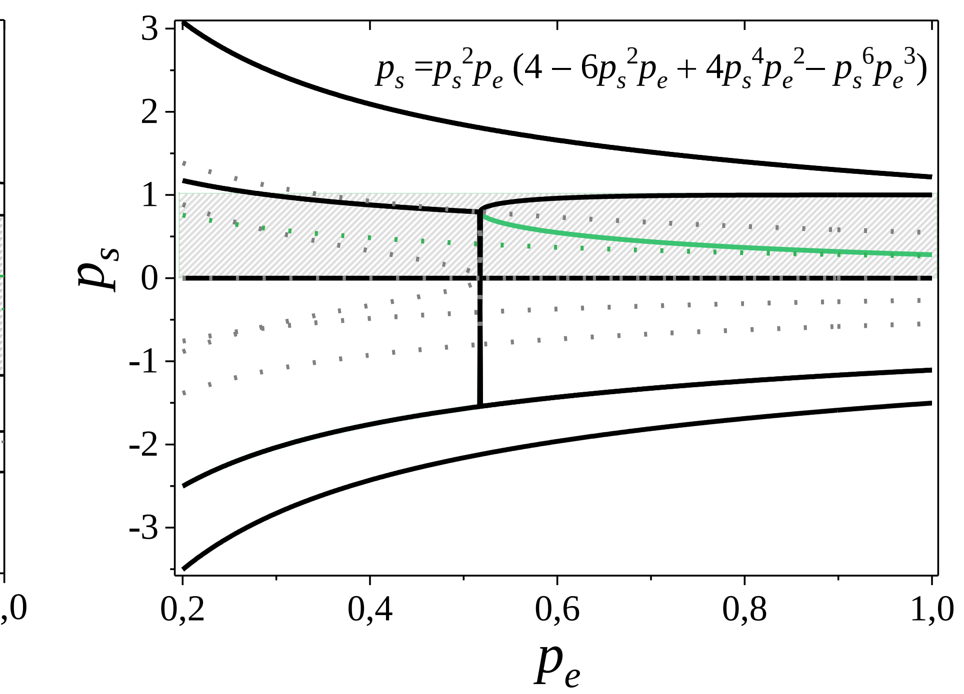
Our extension of renorm-group method

1) We add the probability p_e that there is a conducting connection between neighboring conducting nodes;

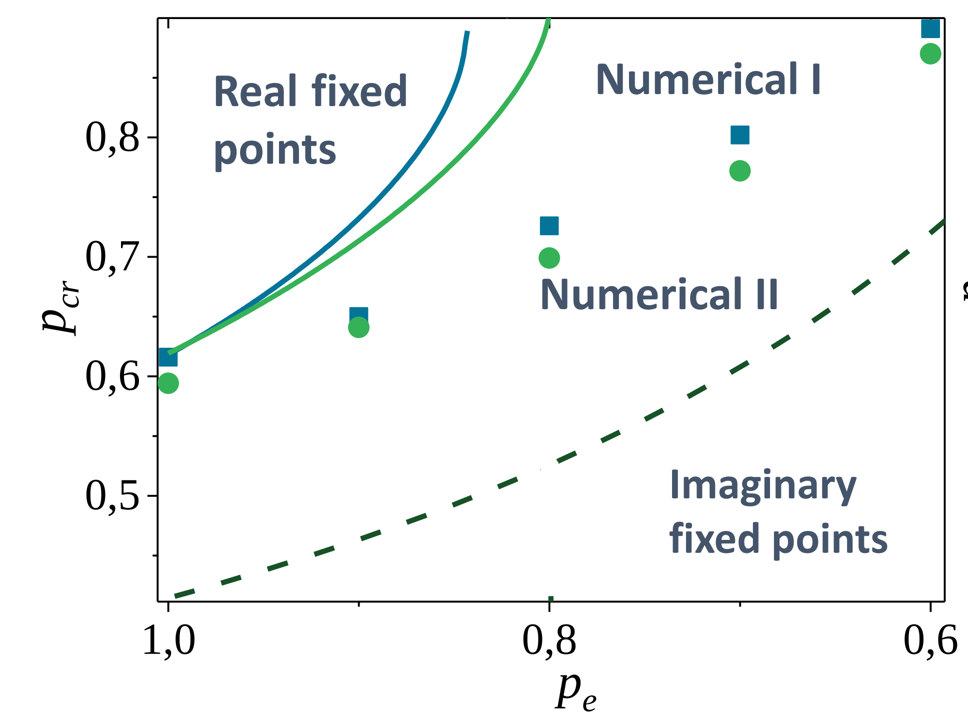
2) One of fixed points of polynomial mapping vs. p_e shows percolation threshold position.



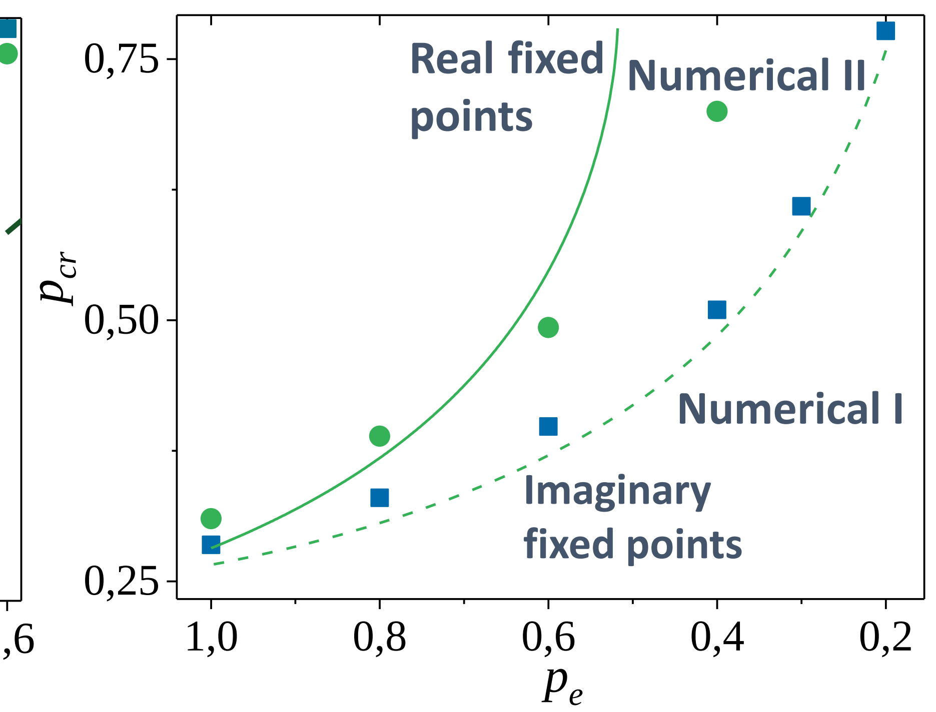
Square lattice 3x3 cell



Cubic lattice 2x2x2 cell



Square lattice



Cubic lattice

Percolation threshold p_{cr} vs p_e

Conclusions

- Two direct modelling approach for mixed percolation studies were developed and tested on square and cubic lattices. Time complexity of both algorithms increases no faster than $O(V^{1.04})$, which means good scalability;
- New analytical estimation approach was developed for mixed percolation problem on the basis of fixed-points analysis of polynomial maps;
- It was established that there is an imaginary fixed point, which limits the percolation threshold from below and real fixed point, which limits the percolation threshold from above.