Aging in the Two-Dimensional Long-Range Ising Model with Power-Law Interactions

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Abstract

The current understanding of aging phenomena is mainly confined to systems with short-ranged interactions. Little is known about the aging of long-ranged systems. Here we present first results of Monte Carlo simulations for the aging in the phase-ordering kinetics of the $d = 2$ dimensional Ising model with power-law long-range interactions $\propto r^{d+\sigma}$. The dynamical scaling of the two-time spin-spin autocorrelator is shown to be well described by simple aging for all interaction ranges studied. The autocorrelation exponents are consistent with $\lambda = 1.25$ in the effectively short-range regime with $\sigma > 1$, while for stronger long-range interactions with $\sigma < 1$ the data are consistent with $\lambda = d/2 = 1$. For very long-ranged interactions, strong finite-size effects are observed. We discuss whether such finite-size effects could be misinterpreted phenomenologically as sub-aging.

Model and Phase Ordering Kinetics

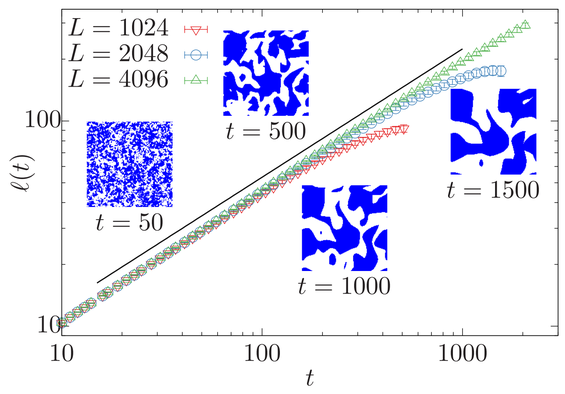
The long-range Ising model with power-law decaying potential can be described by the Hamiltonian

$$\mathcal{H} = -\frac{1}{2}\sum_i \sum_{j\neq i} J(r_{ij}) s_i s_j \text{ and } J(r_{ij}) = \frac{1}{r_{ij}^{d+\sigma}}$$

where the spins $s_i = \pm 1$ are placed on a square lattice.
In phase ordering kinetics, starting from a disordered configuration, this system is then quenched to $T < T_c$ and the ordering of the system is investigated. For this model, there exists a prediction for the characteristic length during this process [1]:

$$\ell(t) \propto t^{\alpha} = \begin{cases} t^{\frac{1}{1+\sigma}} & \sigma < 1 \\ (t \ln t)^{1/2} & \sigma = 1 \\ t^{\frac{1}{2}} & \sigma > 1 \end{cases}$$

For $\sigma > 1$ one thus sees short-range like behavior, for $\sigma < 1$ the growth becomes σ dependent. We have shown this for the first time numerically in Ref. [2].



$\ell(t)$ for $\sigma = 0.6$
solid black line is prediction

Aging

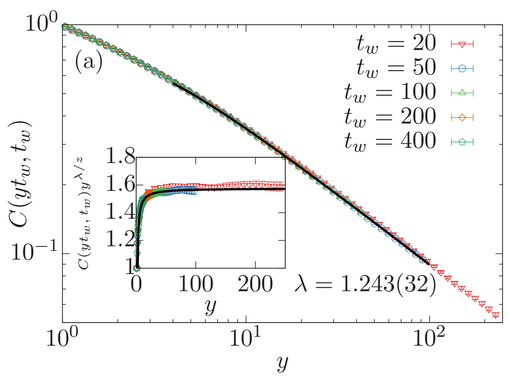
Correlation in time at same position of the local order parameter: $C(t, t_w) = \langle s_i(t) s_i(t_w) \rangle$ with t the observation and t_w the waiting time, where in equilibrium

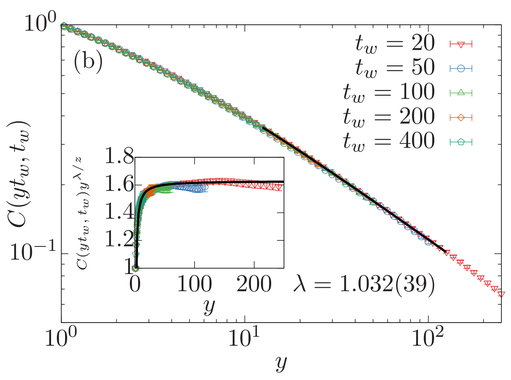
$$C(t, t_w) \sim g(t - t_w)$$

Physical aging has three conditions:

- Slow dynamics
- Loss of time-translational invariance: $C(t, t_w) \neq g(t - t_w)$
- Dynamical scaling: $C(t, t_w) \sim f(y)$, with $y = t/t_w$

Asymptotically for $t \to \infty$ one often has: $C(t, t_w) \sim y^{-\alpha\lambda}$ with the nontrivial aging exponent λ.
We investigate for the first time aging for the long-range Ising model and observe that all three conditions necessary for physical aging are fulfilled [3]:

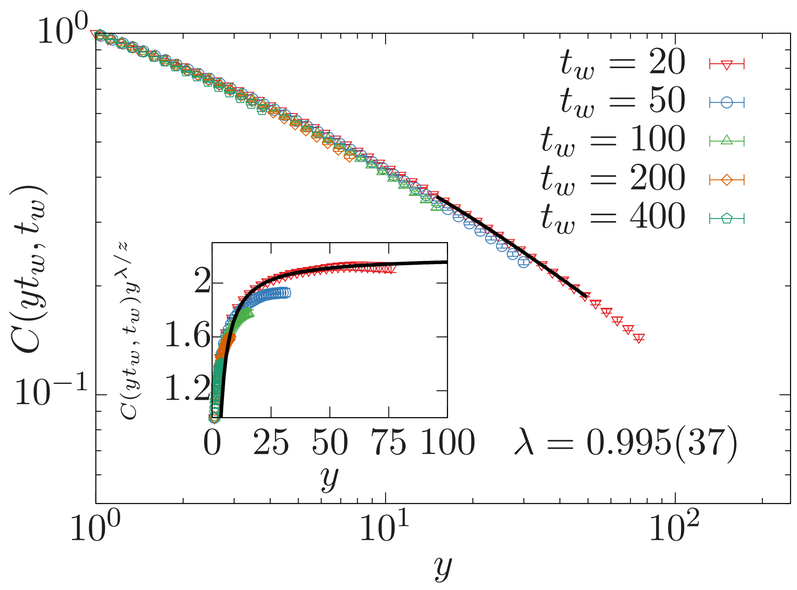




A fit using ansatz $C(yt_w, t_w) = Ay^{-\alpha\lambda}(1 - B/y)$ yields $\lambda = 1.243(32)$ for $\sigma = 1.5 > 1$ in (a), compatible with $\lambda \approx 1.25$ found for the nearest-neighbor model.
For $\sigma = 0.8$, we find $\lambda = 1.032(39)$ in (b), compatible with the rather general lower bound $\lambda = d/2$ [4].

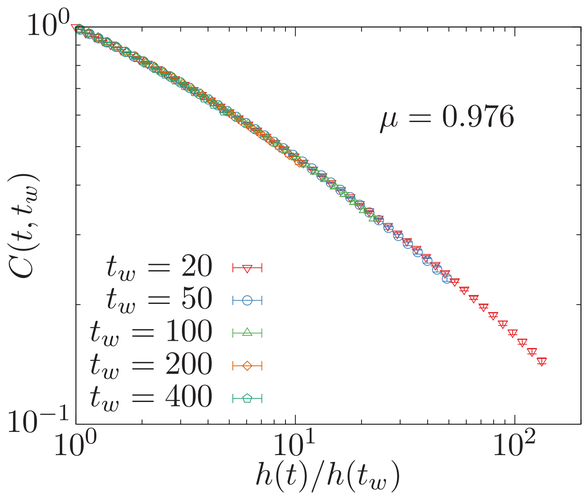
Aging for Small σ

For $\sigma = 0.6$, the dynamical scaling is less convincing:



Possible alternative approach for superior data collapse (also used for glassy systems): Sub-aging with respect to $h(t)/h(t_w)$ instead of t/t_w, where $h(t) = \exp\left(\frac{t^{1-\mu}-1}{1-\mu}\right)$ with the limiting behavior $h(t) \to t$ for $\mu \to 1$

- $\mu > 1$ called super-aging and shown to be impossible [5]
- $\mu < 1$ called sub-aging and observed for example in spin glasses (also experimentally, e.g., for AgMn) [6]



Superior impression of data collapse, BUT with a careful estimation of the onset of finite-size effects one recovers simple aging and again finds $\lambda = 0.995(37) \approx d/2$, compatible with the bound and the data for $\sigma = 0.8$.

References

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Conclusion

We have performed the first numerical investigation of aging in long-range systems by systematically tuning the interaction range using the two-dimensional long-range Ising model. We find for all σ simple aging, where for $\sigma = 0.6$ it is shown that strong finite-size effects may be misinterpreted as sub-aging. The autocorrelation exponent is consistent with $\lambda = d/2 = 1$ for $\sigma < 1$ and with $\lambda = 1.25$ for $\sigma > 1$. This implies that the transition between the short-range and long-range 2D Ising universality classes occurs at a different value of σ than it does either *at* the critical point or else in equilibrium.