Crackling avalanches due to short range interaction in the limit of high disorder

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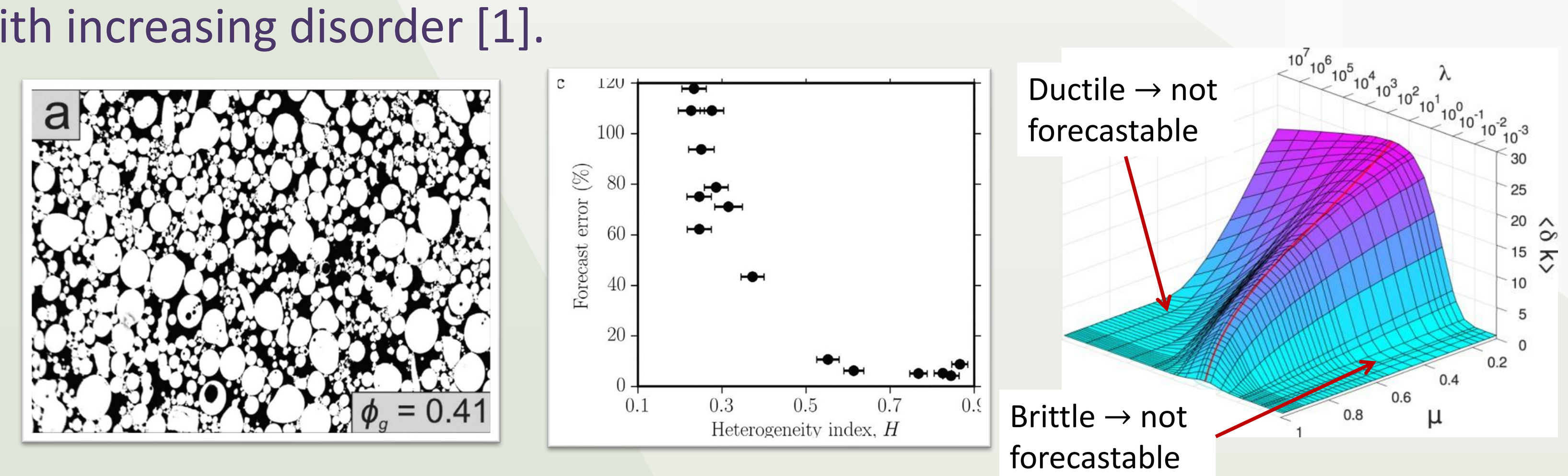
Motivations

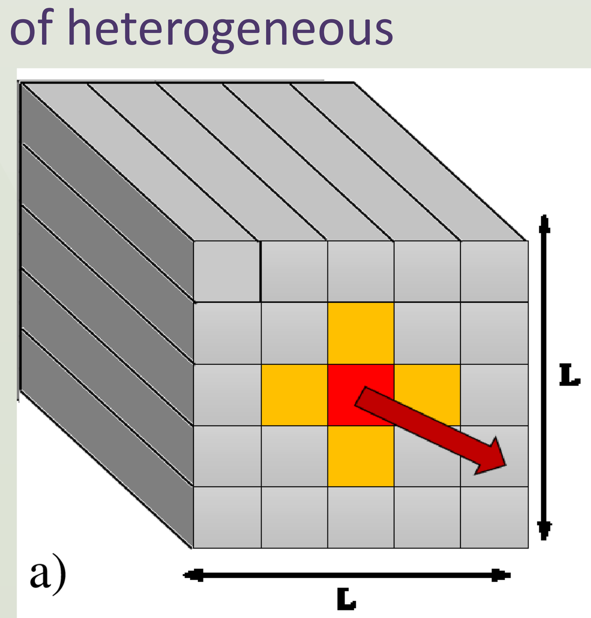
Experiments on the compressive failure of porous glass samples where the degree of heterogeneity could be well controlled during the sample preparation show that the precision of failure forecast methods improve with increasing disorder [1].



Recently, we showed that if a system has a fat-tailed disorder and the interaction is long-range, forecastability is limited to a well-defined degree of disorder: beyond a certain bound, disorder is disadvantageous for forecasting [2]. In solids, however, the range of load redistribution may be limited to the vicinity of failed regions. Hence, here, we investigate the effect of short-range interactions on the forecastability of catastrophic failure.

The fiber bundle model

Our work is based on the fiber bundle model (FBM) of heterogeneous materials [3].



- In FBMs the sample is discretized in terms of parallel fibers
- Fibers:
  - loaded parallel to fibers' direction
  - show perfectly brittle behavior
  - have the same elastic modulus E
  - have random breaking threshold σ_{th}
- Equal load sharing (ELS): The excess load after failure is equally shared by all the intact fibers.
- Local load sharing (LLS): The excess load is shared by the nearest neighbors → complex stress field.

Fat-tail disorder

The extremely high disorder is implemented by using fat-tail distributed breaking thresholds in FBMs [4,5].

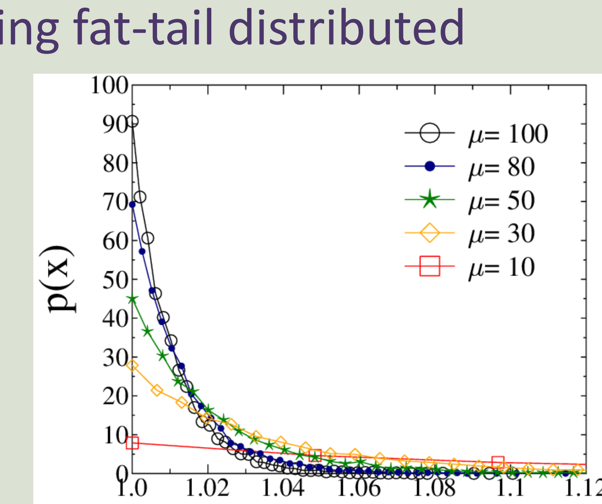
The probability density function :

$$p(\varepsilon_{th}) = A\varepsilon_{th}^{-(1+\mu)} \qquad \varepsilon_{min} \le \varepsilon_{th} \le \varepsilon_{max},$$

where $\varepsilon_{min} = 1$

The amount of disorder is controlled by $\boldsymbol{\mu}$, $\boldsymbol{\varepsilon_{max}}$

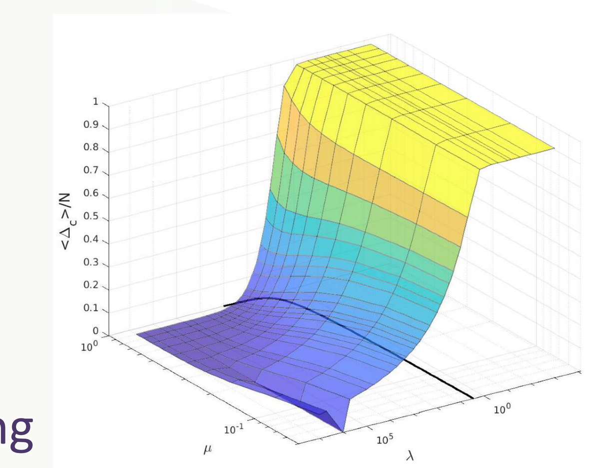
The exponent is varied over the interval $0 \le \mu \le 1$



Phase diagram

Increasing ε_{max} at a fixed μ the system exhibits a transition between two phases:

- **Brittle behavior:** the bundle fails immediately at the first fiber breaking
- **Quasi-Brittle behavior:** macroscopic failure is approached through crackling bursts.
- **Ductile behaviour for $\varepsilon_{max} \to +\infty$**

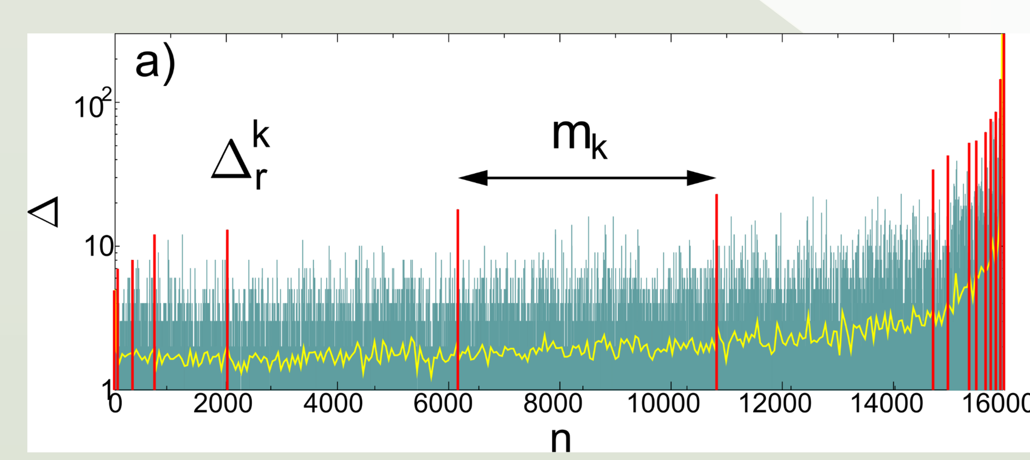


In case of ELS the phase boundary can be determined analytically:

$$\varepsilon^c_{max} = \frac{\varepsilon_{min}}{(1-\mu)^{1/\mu}} \qquad \lambda = (\varepsilon_{max} - \varepsilon^c_{max})/\varepsilon^c_{max}$$

In case of LLS the phase diagram can only be determined numerically, using the size of the catastrophic avalanche.

Statistics of record breaking events



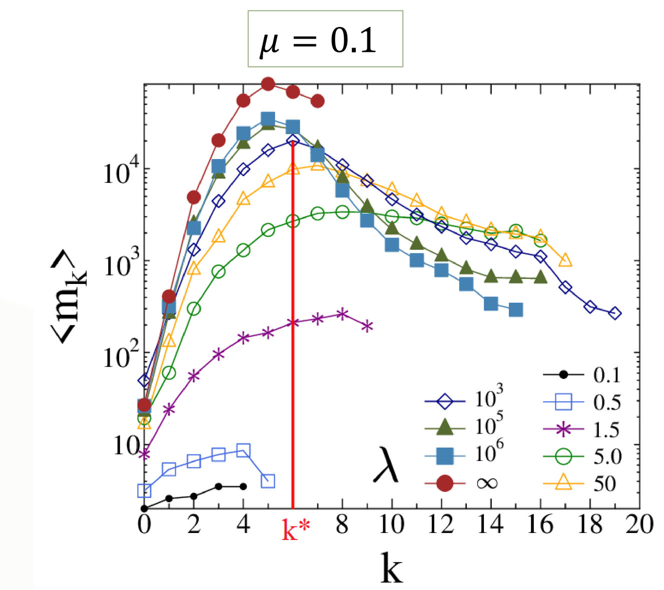
Burst size Δ is defined by the number of fibers breaking in a correlated trail.

Records are bursts which have a size greater than any previous event of the burst sequence [1].

Record breaking events are identified by their rank $k = 1,2, \dots$ which occurred as the n_kth event with size Δ^k_r.

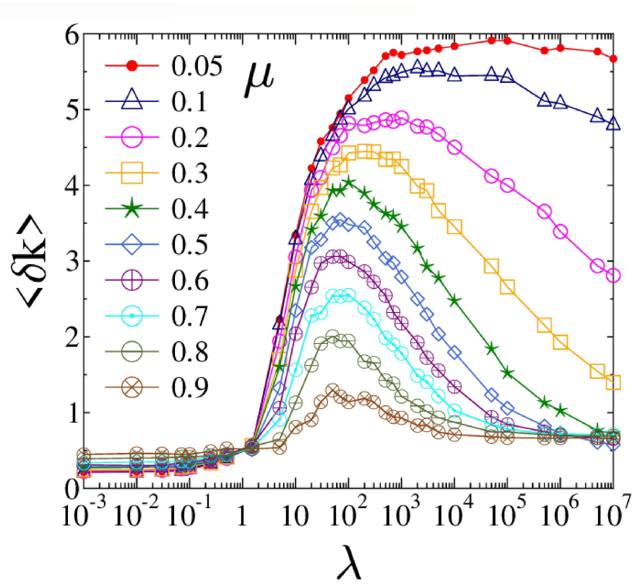
$m_k = n_{k+1} - n_k$ is the waiting time between records (lifetime of records).

Approach to failure through record breaking



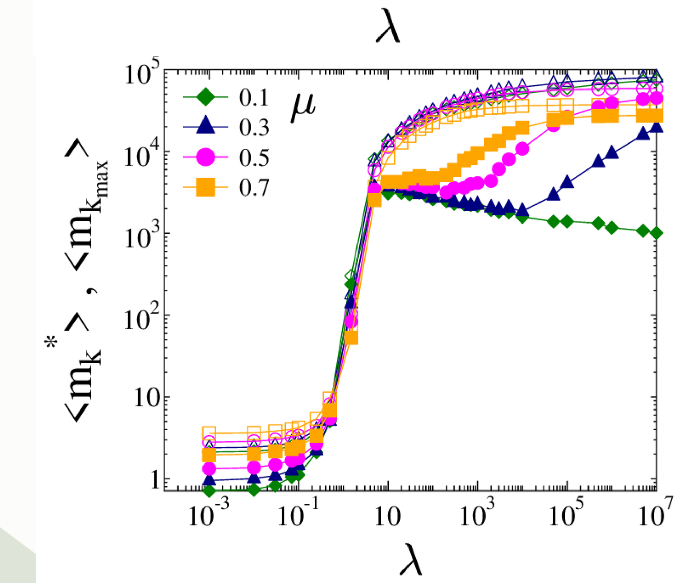
Measuring the average record lifetimes in funcion of the record rank k, for small μ values the same behavior is obtained as for the ELS model:

- Characteristic record rank k^* separates two phases.
- For $k < k^*$ record breaking slows down.
- For $k > k^*$ record breaking accelerates.
- The degree of disorder determines the significance of acceleration

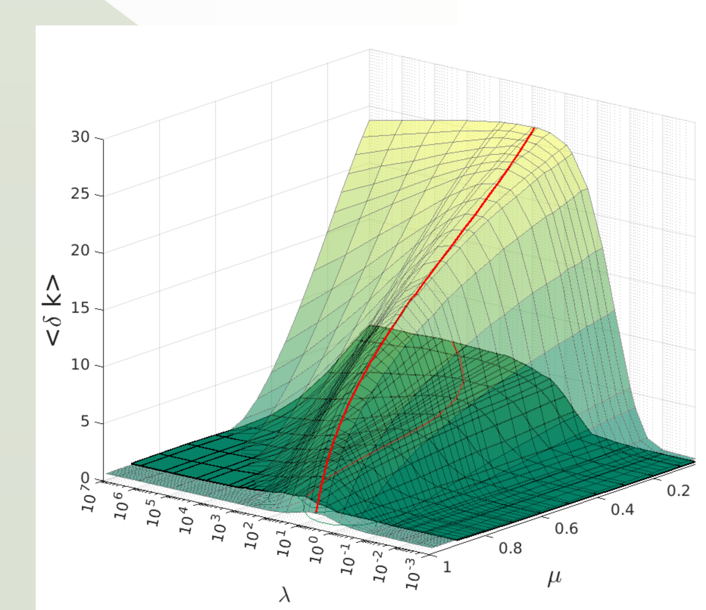


To quantify the extension of the accelerating regime we determined the average difference between the position of the waiting time's maximum and the largest rank $\langle \delta k \rangle = \langle k_{max} - k^* \rangle$. For low disorder (high brittleness) and high disorder (ductile-like behavior) the value $\langle \delta k \rangle \approx 0$ implies no predictability.

Between these two regimes acceleration occurs, but it is too short for predictions.



It is also confirmed by the comparison of the maximum lifetime $\langle m^*_k \rangle$ and the lifetime of the last record $\langle m_{k_{max}} \rangle$.



Conclusions

- For low strength disorder the stress concentration dominates the fracture process → the system is highly brittle, hence, the failure is not predictable.
- At a sufficiently high degree of disorder, global failure is preceded by an acceleration regime.
- This regime gets broader as the amount of disorder increases.
- The acceleration is rather short → forecasting is practically impossible when the load sharing is short ranged.

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